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Tracking error of leveraged and inverse etfs

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TRACKING ERROR OF LEVERAGED AND INVERSE ETFS

by

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ABSTRACT

Tracking ability of leveraged and inverse exchange traded funds can be very important to investors looking for a dependable return. If the investor wants to put their money on a certain index they feel strongly about, they expect their investment vehicle to track that return appropriately. Over the years, we have seen tremendous growth in the exchange traded fund industry. In 2006, leveraged and inverse funds were introduced to the market, allowing investors to take leveraged and directional trades on indices. These investment vehicles can be traded as easily as any stock, and therefore need some attention. Since any novice investor can access and trade these funds, they need to be aware of the risks they are taking.

In this study, I test whether the ProShares S&P tracking leveraged and inverse exchange traded funds track their appropriate index multiple as promised. I did this by running regressions on each fund against the appropriate multiple of their underlying indices. I did this for funds of different market capitalization, for different holding periods, and with different amounts of leverage, to compare how these funds track in different conditions. I found that the large cap funds tend to track the best, with the small cap funds tracking the worst. I also find that tracking error tends to increase with longer holding periods. I find that the distribution of excess returns becomes less normal over longer holding periods, and begins to flatten out and widen. There does not seem to be a concrete conclusion as to whether or not the amount of leverage affects the tracking ability of the funds. I end up with mixed results when comparing amounts of leverage by model fit and by tracking error. Direction also does not seem to play any role in the tracking ability of these funds.

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Chapter One: Introduction

The financial crisis has directed a lot of attention to leveraged products. Jason Zweig in the Wall Street Journal states in 2009, "Leveraged exchange traded funds are the hottest thing on Wall Street. In March alone, $3.4 billion of new money poured into ETFs that use leverage to magnify the returns on U.S. stocks." Leveraging has become more common among investors due to the potential return. Leveraged exchange traded funds (ETFs) have grown significantly over the past five years. As of September 30, 2011, there was $36.6 billion invested in geared ETFs: $11.4 billion in leveraged products and $25.3 billion in inverse funds. With easy access to these higher risk products, investors should be well informed before they invest. On May 2, 2011, Ari Weinberg from the Wall Street Journal wrote, "Although they are mostly targeted to institutions and sophisticated investors, these ETFs trade on exchanges, meaning anyone with an online brokerage account can buy them—as easily as they would a stock." I will look into the risks in this paper; but first, I'll explain the basics.

In financial markets, indices track the performance of a group, such as the prices of equities, exchange rates, or interest rates. The most well-known ones track specific groups of stocks, which, together, are used as benchmarks for the equity market performance. These include the Standard and Poor's 500 (S&P 500), the Dow Jones Industrial Average (DJIA or Dow), and the NASDAQ Composite Index. Each of these tracks a specific part of the overall equity market. The S&P 500, for example, tracks 500 large U.S. companies by market capitalization.

One tool that allows investors to invest in these indices is mutual funds. A mutual fund is an investment vehicle that gathers funds from many investors and invests those funds in securities such as stocks, bonds, money market instruments, or other financial assets. Mutual funds are operated by money managers who invest the fund's capital and attempt to produce capital gains and income for the fund's investors. A mutual fund's portfolio is structured and maintained to match the investment objectives developed by the fund's manager. Although they originated in Europe, the first open-end mutual fund was established in the United States in 1924. Managers would sell shares of the mutual fund to investors, rather than shares of all the individual companies. This created economies of scale in investing, as mutual fund managers could manage the money collectively at a lower cost. Shares of the funds can be created or redeemed by the fund at their net asset value (NAV) per share at the end of each trading day. The per-share value of the fund is calculated by dividing the total value of all the securities in its portfolio, minus any liabilities, by the number of fund shares outstanding.

An index exchange traded fund (ETF) is a security that tracks an index, a commodity, or a basket of assets, but trades like a stock on an exchange. ETFs came about in the late 1980's, but really took off with the State Street Global Advisors S&P 500 tracking ETF, the spider (ticker symbol: SPY). SPY was first traded in 1993, and it is now the largest ETF in the world with more than 300 hundred million shares traded each day and over 81 billion dollars in assets as of October 1, 2011.

ETFs track a wide variety of assets. The most common ETFs track well known indices such as the S&P 500, the Dow Jones Industrial Average, the NASDAQ, and the Russell 2000.

They also track other equity market sectors, commodities, bonds, stock price volatility, and much more. ETFs give investors easy access to baskets of assets.

It is important to distinguish between primary and secondary markets when studying ETFs. A primary market is one in which new securities are issued, such as when companies, governments, and other entities obtain financing through issuing debt or equity securities. Primary markets are facilitated by underwriters, typically investment banks, which help set the issue price for a security and assist in selling it to investors. A secondary market is one in which investors purchase existing securities or assets from other investors. The national exchanges, such as the New York Stock Exchange and the NASDAQ, are secondary markets. In any secondary market transaction, the cash proceeds go to an investor rather than the issuer of the security. Trading of ETFs occurs in the secondary market.

While both mutual funds and ETFs allow investors to invest in a portfolio of assets, there are differences between them. Shares in ETFs can be sold short. This is important for investors who, not only pull out their funds in down markets, but also want to profit from the downturn. Short selling requires borrowing shares from a broker, selling them in the open market, and buying them back in the future and returning them to the broker, hopefully at a lower price. The investor's profit is the difference in price, minus any transaction costs and dividends. Investors cannot short mutual funds because their shares are not sold in a secondary market. An investor who wants to "sell" shares in a mutual fund redeems them with the fund provider.

Another advantage of ETFs that has been debated is their tax efficiency. ETFs are more tax efficient because they do not create realized capital gains when shares are issued or

redeemed. When investors redeem mutual fund shares, the fund's manager might have to sell securities in order to generate cash for the redemption. Such selling might create realized capital gains or losses, which are distributed to the fund's remaining investors. If an investor owns a mutual fund when it sells securities it is holding, he or she is responsible for paying taxes on any capital gains, even if he or she did not earn those gains. When invested in an ETF, you only pay taxes on the gain or loss in share price and dividend distribution that is realized when the shares are sold.

Another advantage of ETFs is the ability to trade intraday. With most open end mutual funds, an investor can only acquire or redeem shares at the end of the day, at that day's closing NAV. Others can only be acquired or redeemed at set times throughout the day, but still not at any instance. If an investor anticipates a move in the market and wants to trade on it, they will have to wait until the set time or the market close. ETFs trade just like a stock, so an investor can buy and sell at any point during the day while the market is open.

Although it seems that exchange traded funds are better investment vehicles mutual funds, they have a few disadvantages. One of these involves transparency. ETFs must provide complete transparency to the public regarding assets held and traded. When an index releases information stating it will be rebalancing its portfolio, investors can anticipate which shares will be bought or sold, creating a front running opportunity. Front running is buying (selling) a security before the large fund does, to capture the increase (decrease) in price that is very likely to occur when the fund buys (sells) a large volume of the security.

An issue that affects both mutual funds and ETFs is the way they reinvest their dividends. Most funds don't reinvest their dividends as they come in. This would not be efficient due to trading costs. Instead, they pool the cash flows, and reinvest at set time intervals, or not at all. Some funds pay out cash inflows to investors as dividends. The result, in a rising market, is underperformance to the underlying index. Of course, this can help the fund outperform the index in a declining market.

Another issue that can affect both index mutual funds and index exchange traded funds is tracking error. Index mutual funds and index ETFs are designed to mimic (track) a specific underlying index. An index mutual fund is always invested in or redeemed at NAV so its performance will only deviate from the index if its holdings fail to match the index. In contrast, an index ETF trades like a stock, so its price may deviate from its NAV. Investors can create new shares of the ETF if the price is higher than the NAV, or redeem shares if the price is lower than the NAV, in order to bring the fund back to its NAV. This is not always feasible due to the transaction costs involved. In either case, these deviations can result in what is known as tracking error. Tracking error is the focus of this thesis.

Investors can amplify returns by leveraging. Leveraging is using borrowed capital to purchase additional assets. Derivatives are securities that derive their value from an underlying asset, and allow the investor to leverage their position. Leverage and Inverse ETFs use derivatives to achieve this leverage and enhanced returns. Inverse ETFs are geared exchange traded funds give the investor the negative of the return of the underlying index. These can be leveraged as well. Inverse ETFs are used to place a bet on a downturn in the market. Leveraged

ETFs started appearing in 2006, and have growth substantially ever since. As of September 2011, there were $36.6 billion dollars of assets under management in leveraged and inverse ETFs. They use various financial instruments, such as derivatives, or borrowed capital, such as margin, to increase the potential return of an investment. Some leveraged ETFs promise investors two or three times the return of an underlying index, while other inverse ETFs promised one, two, or three times the negative e of an index return.

The funds allow investors to make oversized bets on the market, long or short, by using swaps, futures, and other derivatives," says Murray Coleman of Barrons. A typical two times leveraged ETF seeks to track two times the return of the underlying index. The concept is simple; an investor buys the 2x leveraged ETF, Ultra S&P 500 (Ticker: SSO), and expects to earn twice the return of the S&P 500. Inverse ETFs use short selling and leverage to create the negative of the return of their underlying index. An investor should therefore be able to buy the 2x inverse (-2x) ETF, Ultra Short MidCap 400 (Ticker: MZZ), and expects to earn twice the negative of the return on the S&P 400. This should hold true in theory, but these leveraged products typically have much higher levels of tracking error than traditional (1x) index ETFs.

Tracking error is the divergence between the price of a portfolio and the price of its benchmark, or underlying index. There are many ways to compute tracking error. One way to calculate tracking error is the root mean squared error (RMSE). This is done by squaring each deviation (the difference between the portfolio return and the benchmark return), calculating the arithmetic average of the squared deviations, and then calculating the square root of that average.

Another way of calculating it is as the standard deviation of the percent difference between the fund return and the index return. This is the way I will calculate tracking error in my analysis.

It is also important to note that tracking error can be measured as ex-ante or ex-post. Ex-ante tracking error is a forecast of what tracking error is expected to be. Ex-post is a calculation of tracking error based on historic returns. In this thesis, the focus is on ex-post tracking error.

Investors have different opinions about tracking error. It is most often viewed negatively because it indicates the fund manager is not doing what was promised; that is, not tracking the index (underlying asset prices), although a small amount of tracking error may be unavoidable and is therefore accepted. Even if the tracking error is associated with an average excess return above the underlying index (positive alpha), giving the investor better returns, it can still be seen as bad. Problems may arise between a manager and investors if the manager is not tracking an index the way promised in the ETF prospectus. Investors do not want excess risk being taken with their money.

On the other hand, some investors take a more optimistic approach when looking at tracking error. They believe that if there is positive alpha, there is no reason to complain about the associated tracking error. Of course, this only holds true when the manager is creating positive alpha, not negative.

Most exchange traded funds don't track their underlying portfolio perfectly. Tracking error can come from a variety of sources including commissions, rebalancing, and transaction costs. Fund management fees can also cause an investor's return to be less than the benchmark. Managers rebalance their portfolios periodically to align them better with the underlying index or

portfolio of securities. This can happen because the index changes the weights or securities held in it. Rebalancing the portfolio re-aligns the weights of the assets in the portfolio with those in the underlying index. This does not happen continuously; therefore, the fund may not, at every instant, hold an exact replica of the underlying index. This can cause returns to deviate from the index. The final major contributor to tracking error is commissions. When managers trade, possibly for rebalancing, they incur transaction costs. These costs, although maybe small, are not found in the index returns. For these reasons, some small amount of tracking error would be expected even for the best run funds.

Because of the techniques used to create leveraged and inverse ETFs, tracking error can be even more severe for them. "The zigs and zags of daily price action can cause leveraged ETFs to chart unexpected courses," says Marc Gerstein in Forbes (2010). Leveraged ETF managers will not just try to attain a representative sample of the underlying portfolio, as this will only give them the same return (roughly). Instead, they may use derivatives, margin, or other means to create the multiplied return of the index. A typical leveraged ETF will hold a small portion of swaps, options, and futures, and other derivative contracts, and a larger portion of cash. The cash is held to manage losses on the derivative contracts. This technique can cause the tracking error to be larger than in traditional funds.

There has been a lot of research done regarding ETFs, some of which has been focused on leveraged and inverse products. Researchers have looked at the price behavior and tracking ability of these funds. In this paper, I will be looking at the tracking ability of the world's largest

manager of leveraged and inverse ETFs, ProShares, in multiple ways to understand what causes greater amounts of tracking error.

Chapter Two: Literature Review

Investors are concerned with tracking error because it impacts their returns. Poterba and Shoven (2002) look at a comparison of Index Fund and ETF returns on a pre-tax and after-tax basis. They find that there are many contributors to deviations of returns of each to the underlying. These include expense ratios, purchase price (or purchase value), and tracking error on a pre-tax basis. Deviations are further realized on an after-tax basis when the investor accounts for the difference in realized capital gains between the funds. They also find that the Vanguard 500 Index fund outperformed the market in the sample period 1994-2000, net of expenses. They believe this is attributable to the Vanguard fund rebalancing when the index announces it will. The index announces rebalancing before it actually does. This allows the fund to gain from the buying of the rebalanced portfolio.

Robertson (2003) also looks at what causes tracking error in equity index funds. He believes that the goal of index funds is to minimize both expense ratios and tracking error, and that funds should be evaluated on these two metrics. He finds that tracking error is attributable to the use of derivatives, daily volatility, sampling error, and liquidity issues. First, funds will use derivatives to hedge prices until they can buy or sell a portion of their portfolio. The use of derivatives is costly, therefore creating tracking error. Next, funds may buy or sell securities throughout the day, and not at their closing price. This can cause the fund's daily price to deviate from the index. Further, some indices contain a large number of securities. Having to fully replicate these indices would be costly. Instead, fund managers may hold a representative group

of the securities, known as a sampling approach to index replication. The sampling error associated with not holding exactly the same securities as the index leads to more tracking error. His final variable is liquidity issues. Not all securities are traded every day. Smaller stocks may not have the volume to trade, or trading them may cause substantial price changes. This is called market impact cost. He believes these three issues cause substantial tracking error in index funds.

Gastineau (2001) provides good background on ETFs and how they originated. He discusses products that came before ETFs, such as program trading or portfolio trading. This is the original way investors traded portfolios of stocks, like the S&P 500, in a single trade. He also reviews the redemption and creation in ETF shares. These features allow for arbitrage to keep the fund trading close to its NAV.

Elton (2002) studies how dividend reimbursement to investors affects the returns of ETFs compared to their underlying index. He finds that if the fund decides to hold the dividends to release them in set intervals, say quarterly, the fund will underperform the index in a rising market, if the index treats dividends as being reinvested immediately. This is due to the fund missing out on the gains that could be earned on reinvested dividends in a rising market.

Aber, Li, and Can (2009) look at the tracking ability for iShares ETFs. They find that most ETFs trade at a premium to their NAV, even on high volume days (where arbitrage should be easy), suggesting the market overvalues ETFs. They also find that ETFs and mutual funds vary when compared by tracking error. Some mutual funds and ETFs have almost identical

returns. Others differ by more than ten percent, with the ETF performing better than the mutual fund. This shows that tracking is an issue for most fund types, and there is reason to examine it.

Rompotis (2011) looks at tracking error by comparing 50 iShares funds' performances with the performance of the underlying indices. He states, "When we use the NAV return tracking errors, we remove the expense ratio from the model because NAVs are free from management expenses and therefore there must be no sensible relationship between tracking error and expense ratio." He finds that the age and risk of the fund directly and significantly affect its tracking error. He also finds that funds can be constructed in a variety of ways, using a mixture of growth and value, large cap and small, emerging markets and established, etc., to have positive alphas. They have higher risk-adjusted returns when expressed by their Sharpe ratios.

In 2006, leveraged and inverse ETFs were first released into the market. Trainor and Baryla (2008) ask whether leveraged ETFs provide the returns they promise, and study the long run holding period returns of leveraged ETFs. They find that returns vary in different market conditions. They demonstrate that returns on leveraged ETFs are lognormally distributed. As they state, "Although it is a statistical fact that compounding random returns causes long term returns to be lognormally rather than normally distributed, the effect of this is not always understood." This is important because it causes the distribution of returns to be positively skewed. With a lognormal distribution, the median is below the mean. This tells us that investors are statistically more likely to get a return less than the average return; however, the returns that are larger than the average can be a lot larger. Another important concept they

discuss is the constant leverage trap. This is not a new concept, and has been talked about by fund companies and in the popular press. They state:

> Constant leverage requires an investor to maintain an exact percentage of leverage over the entire time horizon. If one is using a margin account, this requires an investor to buy in a rising market and sell in a declining market. The 'trap' occurs because this type of strategy magnifies the compounding problem. The compounding problem is based on the mathematic principle that the geometric mean of a series of numbers is lower, the greater the variance of the numbers. Using leverage magnifies the variance of the returns. (Trainor and Baryla, 2008).

Modeling both concepts, they run Monte Carlo simulations on leveraged ETF returns. They find, for holding periods out to ten years, a typical 2x leveraged ETF only returns 1.4x the index on an annual basis; however, an investor still assumes twice the risk of the traditional ETF measured by the standard deviation of returns. They also compare this with buying a traditional ETF on margin as a way of replicating a leveraged ETF. Their results show leveraged ETFs are better due to lower costs of trading and avoiding interest on the margin account.

Militaru and Dzekounoff (2010) compliment this research, finding that the amount of volatility determines the distribution of possible returns for a leveraged fund, using the fund SKF for their analysis. They state, "The lower the volatility, the more symmetric the alternative outcomes will be." They find that high volatility pushes the majority of possible returns downward.

Charupat and Miu (2011) find that, while price deviations are generally small, the leveraged and inverse ETFs are more inclined to large price premiums or discounts. These deviations are generally larger than with traditional ETFs. They also find that while bull leveraged ETFs (leveraged ETFs structured to return positive multiple returns of the index) trade

at a discount or slight premium, on average, bear leveraged ETFs (leveraged ETFs structured to return negative multiple returns of the index) tend to trade at relatively larger premiums. They state, "This is consistent with the fact that premiums occur more frequently than discounts for all bear ETFs. In contrast, we observe discounts more frequently than premiums for all bull ETFs."

They also observe that leveraged ETFs successfully deliver the promised return for holding periods of up to one week. However, for holding periods out to one month, there is significant tracking error, especially for inverse ETFs. Holding periods longer than this can see returns considerably different from what is promised.

Chapter Three: Data and Methods

In this paper, I will examine the tracking error of leveraged and inverse ETFs compared to the appropriate multiples of their underlying indices. I will use the ProShares leveraged exchange traded fund series in my research. I will be looking at three underlying indices: S&P400, S&P 500, and S&P 600. The S&P 500 represents the large-cap sector of the U.S. equity market, the S&P 400 represents the mid-cap sector, and the S&P 600 represents the small-cap sector. I will use the ETFs summarized in Tables 1-3 below. Table 1 is the S&P 500 tracking funds. Table 2 is the S&P 400 tracking funds. Table 3 is the S&P 600 tracking funds.

Table 1: S&P 500 Tracking Funds

S&P 500 Funds		
Fund Name	**Ticker**	**Leverage**
UltraPro S&P500*	UPRO	3x
Ultra S&P 500	SSO	2x
Short S&P500	SH	-1x
UltraShort S&P500	SDS	-2x

* Inception Date June 23, 2009

Table 2: S&P 400 Tracking Funds

S&P 400 Funds		
Fund Name	**Ticker**	**Leverage**
Ultra MidCap 400	MVV	2x
Short MidCap 400	MYY	-1x
UltraShort MidCap400	MZZ	-2x

Table 3: S&P 600 Tracking Funds

S&P 600 Funds		
Fund Name	**Ticker**	**Leverage**
Ultra SmallCap 600**	SAA	2x
Short SmallCap 600**	SBB	-1x
UltraShort SmallCap 600**	SDD	-2x

** Inception Date January 23, 2007

There are fewer observations for the S&P 600 tracking funds and the 3x S&P 500 tracking fund because they did not exist until January 23, 2007 and June 23, 2009, respectively. I will run regressions individually on each fund for daily, weekly, and monthly returns. If the leveraged and inverse funds provide what is promised, the alpha (intercept) and beta (slope) estimates from each regression should be 0 and 1, respectively.

I will use historical closing price data from Yahoo Finance for the time period January 1, 2007 to December 31, 2011. I will be using the adjusted close for both the underlying indices and the leveraged ETFs. This allows me to examine an equal amount of data for all of the funds, except for those with inception dates after January 1, 2007. I will use the following measure of tracking error. Excess Return will be the deviation of return from the underlying index. This will be calculated as: $ER = (M \times R_I) - R_{ETF}$, where M is the appropriate multiple, R_I is the index return, and R_{ETF} is the ETF return. Tracking error will be measured as the standard deviation of Excess Return over time. Using this measure of tracking error, and the funds described above, I will examine the effects of leverage on tracking error. I will examine daily tracking error for the time period stated, paying attention to market conditions within the time period. I will also compare this tracking error across the different funds to examine the effects from the amounts of

leverage. By comparing the funds described, I will also be able to see if there are differences in tracking error related to market cap. Finally, I will examine tracking error based on the length of holding period by comparing the daily, weekly, monthly, annual, and cumulative tracking error.

Chapter Four: Results

Daily Returns

The results for the daily return regressions are shown in Table 4 below

Table 4: Daily S&P 500 Fund Regressions

Multiple	No. of Observations	Alpha (P-Value)*	Beta (P-Value)*	Adjusted R-Squared
2X	1260	-0.0001	0.9572	0.9845
		0.2189	0.0000	
3X	635	0.0000	0.9906	0.9918
		0.7949	0.0086	
-1X	1260	0.0001	0.9803	0.9789
		0.2985	0.0000	
-2X	1260	0.0001	0.9652	0.9867
		0.4414	0.0000	

*P-Values for alphas are for testing against an alpha of zero. P-Values for betas are for testing against a beta of one.

Using p-values to measure statistical significance in my analysis, I find that, for the S&P 500 tracking funds, the alphas are not statistically different than zero. The p-values for the alphas: .2189, .7949, .2985, and .4414 for the 2x, 3x, -1x, and -2x funds respectably, are all greater than the .05 significance level. This does not allow me to reject that the null that the alphas are zero. All of the p-values for the betas are less than .05, allowing me to reject the null hypothesis that the betas of the funds are one at the 95% confidence level. They are small enough to reject the null hypothesis even at the 99% confidence level. The adjusted R-squareds for the models are .9845, .9918, .9789, and .9867 for the funds 2x, 3x, -1x, and -2x respectably. This shows that the models are a good fit for the data, as I expected.

The results for the S&P 400 funds are shown in Table 5 below.

Table 5: Daily S&P 400 Fund Regressions

Multiple	No. of Observations	Alpha (P-Value)*	Beta (P-Value)*	Adjusted R-Squared
2X	1260	-0.0001	0.9615	0.9756
		0.5893	0.0000	
-1X	1260	-0.0001	0.9275	0.9098
		0.6242	0.0000	
-2X	1260	0.0001	0.9583	0.9791
		0.7189	0.0000	

*P-Values for alphas are for testing against an alpha of zero. P-Values for betas are for testing against a beta of one.

Again examining the p-values, I find that, for the S&P 400 tracking funds, the alphas are not statistically different than zero, again. The p-values for the alphas: .5893, .6242, and .7189 for the 2x, -1x, and -2x funds respectively, are all greater than the .05 significance level. This does not allow me to reject the null that the alphas are zero. All of the p-values for the betas are less than .01, again allowing me to reject the null hypothesis that the betas of the funds are one at the 99% confidence level. The adjusted R-squareds for these models are .9756, .9098, and .9791 for the 2x, -1x, and -2x funds respectably. This shows that the models are also a good fit for the data, as expected.

The results for the S&P 600 funds are in Table 6 below.

Table 6: Daily S&P 600 Fund Regressions

Multiple	No. of Observations	Alpha (P-Value)*	Beta (P-Value)*	Adjusted R-Squared
2X	1244	0.0000	0.6347	0.5666
		0.9522	0.0000	
-1X	1244	-0.0003	0.6251	0.4524
		0.5420	0.0000	
-2X	1244	0.0000	0.6376	0.5763
		0.9761	0.0000	

*P-Values for alphas are for testing against an alpha of zero. P-Values for betas are for testing against a beta of one.

As before, I cannot reject that the alpha is equal to zero for any fund. The p-values for the alphas are all larger than .05. For the betas, the p-values are all zero, allowing me to reject the null that the betas are one. Looking at the fit of the model, the adjusted R-squareds are only .5666, .4524, and .5763 for the 2x,-1x, and -2x funds, respectably. This shows that the models are not particularly good fits for the data. This tells us that these S&P 600 (small cap) ETFs do not track their index multiples as promised. Looking at the summary statistics for the excess return in Table 7 below, I find that there are some days when the fund produces large excess returns.

Table 7: Fund Daily Excess Returns

Fund	Observation	Mean	Std. Dev.	Min	Max
S&P 500 Funds					
2x Leverage ETF (SSO)	1260	-0.0150%	0.4258%	-6.6490%	3.1240%
3x Leveraged ETF (UPRO)	635	-0.0054%	0.3398%	-3.4700%	3.1590%
-1x Inverse ETF (SH)	1260	0.0073%	0.2428%	-2.1000%	3.7030%
-2x Inverse Leveraged ETF (SDS)	1260	0.0090%	0.3926%	-4.0630%	6.6100%
S&P 400 Funds					
2x Leveraged ETF (MVV)	1260	-0.0112%	0.6018%	-5.9160%	5.1230%
-1x Inverse ETF (MYY)	1260	-0.0056%	0.5771%	-14.4890%	3.9310%
-2x Inverse Leveraged ETF (MZZ)	1260	0.0079%	0.5602%	-5.0330%	5.2020%
S&P 600 Funds					
2x Leveraged ETF (SAA)	1244	-0.0318%	3.1458%	-75.7180%	46.5730%
-1x Inverse ETF (SBB)	1244	-0.0141%	1.8538%	-33.4870%	37.4660%
-2x Inverse Leveraged ETF (SDD)	1244	0.0246%	3.1050%	-46.3600%	75.1940%

The ranges of the excess returns for the S&P 600 tracking funds are a lot larger than the ranges of the excess returns for the S&P 400 and S&P 500 tracking funds. The S&P 600 tracking funds have produced excess returns between -75.718% and 75.194%. These are, however, outliers in the data. By comparison, the S&P 400 tracking funds only have excess returns ranging from -14.489% to 5.202%. The S&P 500 funds are even tighter with excess returns ranging from -6.649% and 3.703%. This is also seen in Figures 1-10 below.

Figure 1: 2x S&P 500 Tracking Fund's Daily Excess Returns

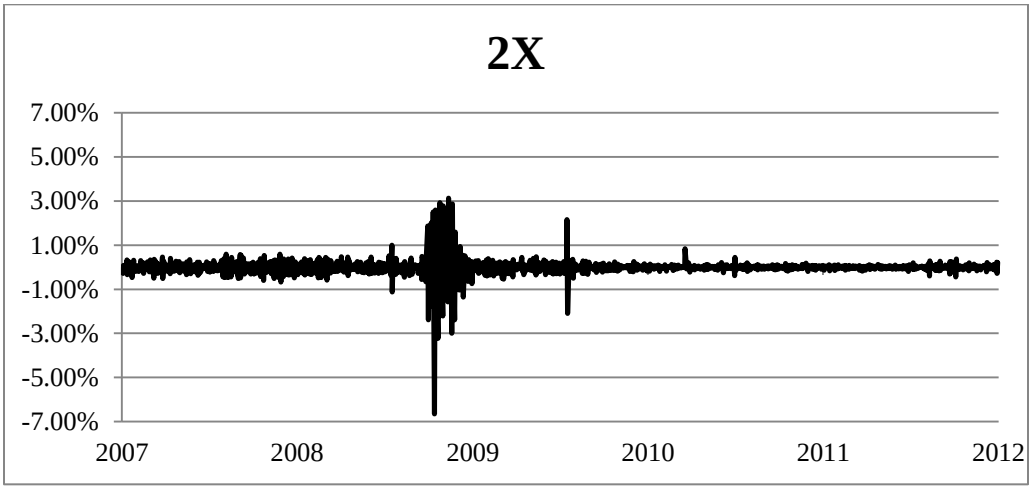


Figure 2: 3x S&P 500 Tracking Fund's Daily Excess Returns

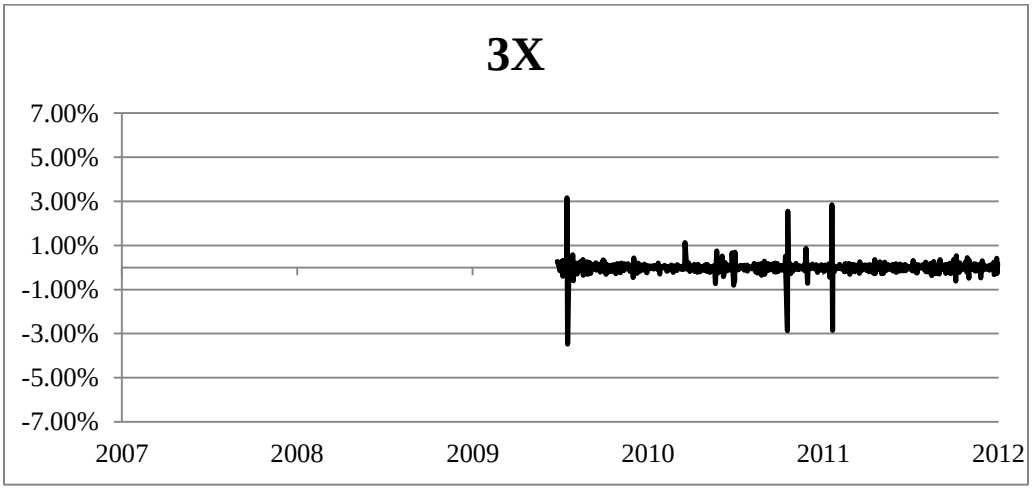


Figure 3: -1x S&P 500 Tracking Fund's Daily Excess Returns

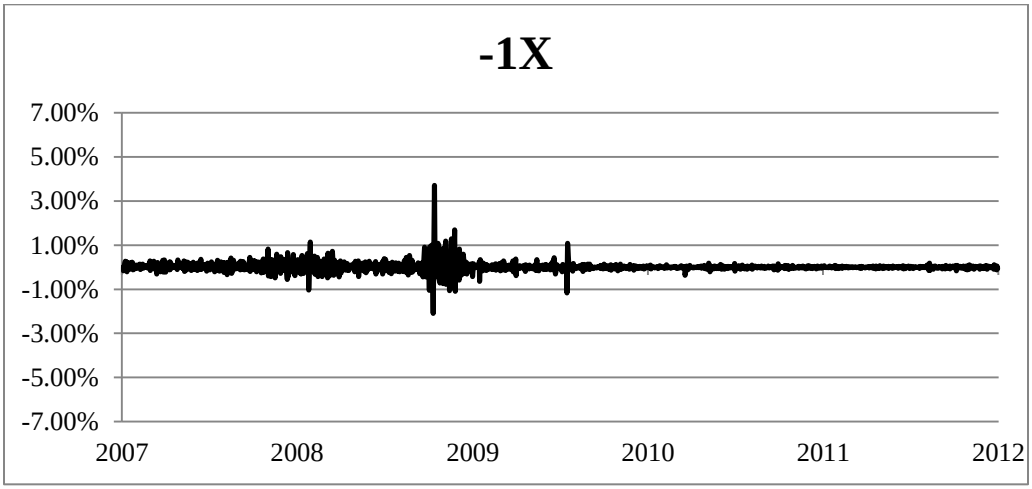


Figure 4: -2x S&P 500 Tracking Fund's Daily Excess Returns

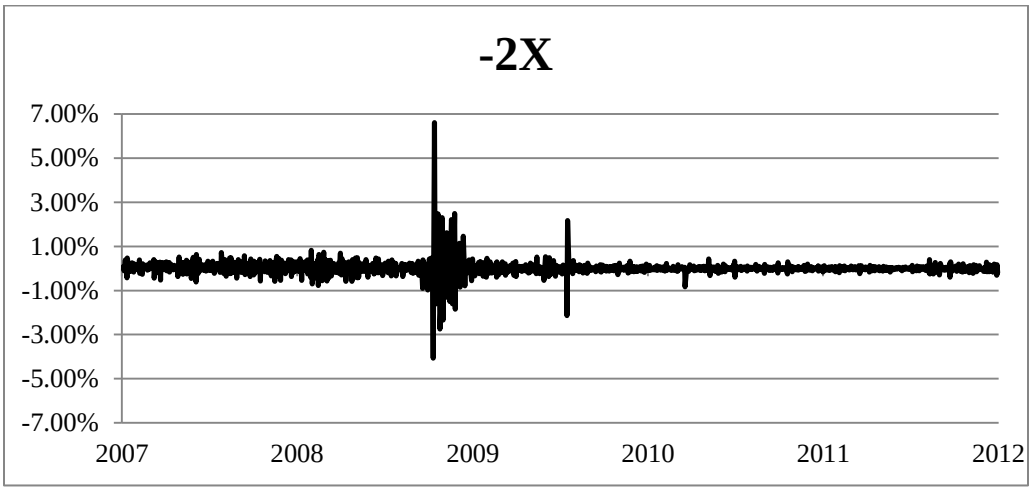


Figure 5: 2x S&P 400 Tracking Fund's Daily Excess Returns

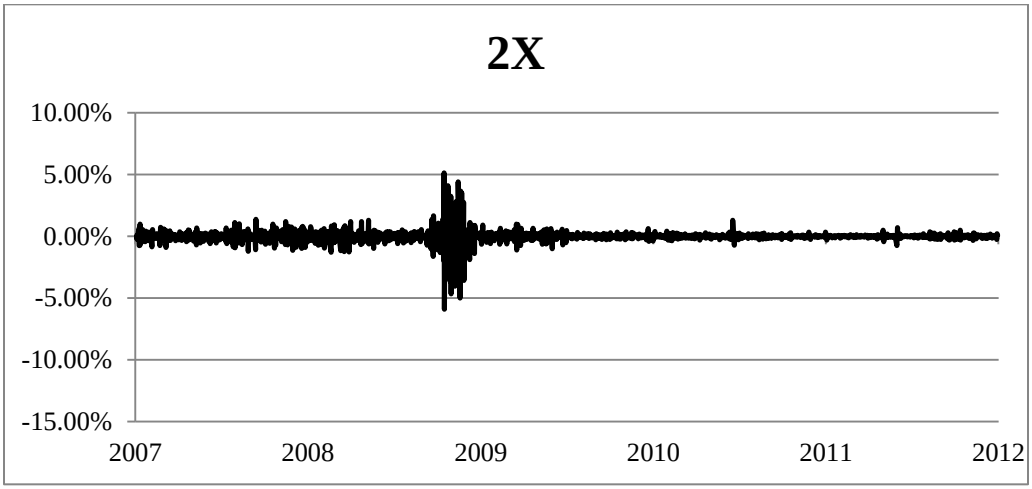


Figure 6: -1x S&P 400 Tracking Fund's Daily Excess Returns

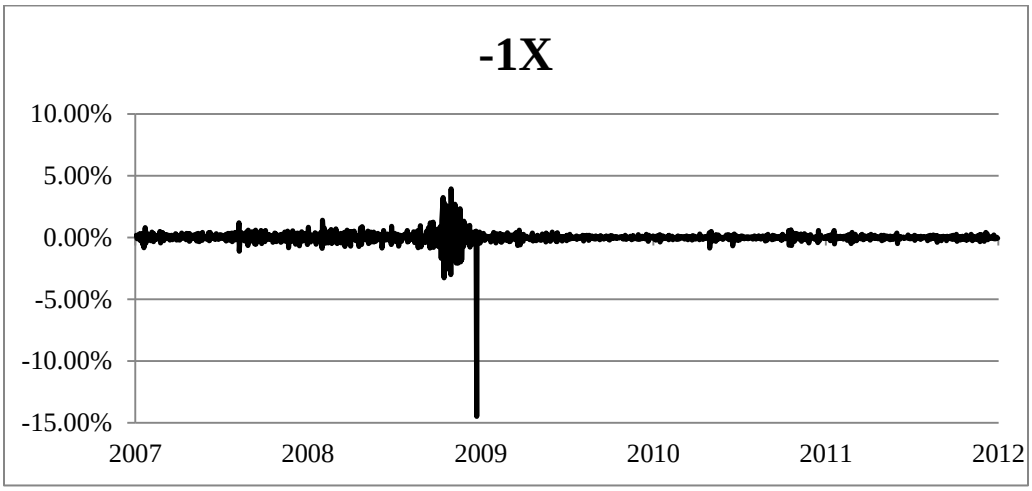


Figure 7: -2x S&P 400 Tracking Fund's Daily Excess Returns

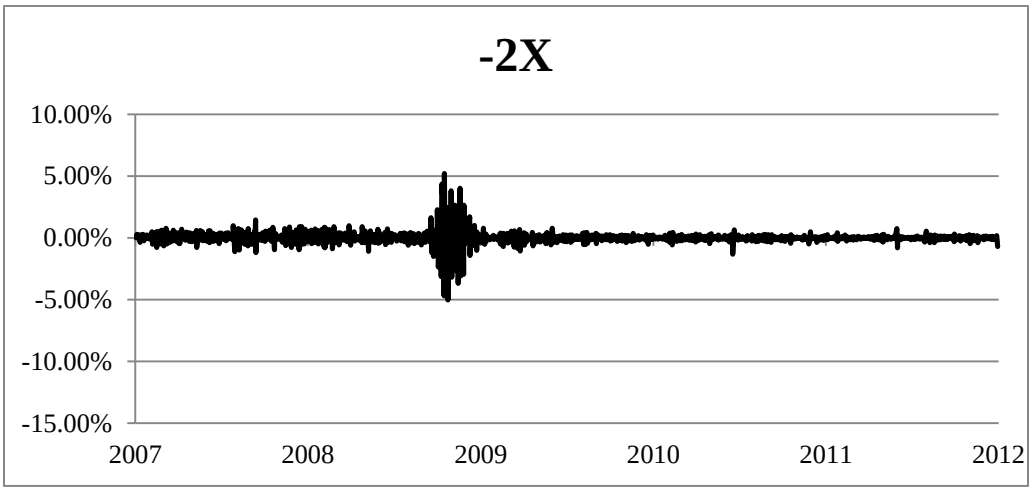


Figure 8: 2x S&P 600 Tracking Fund's Daily Excess Returns

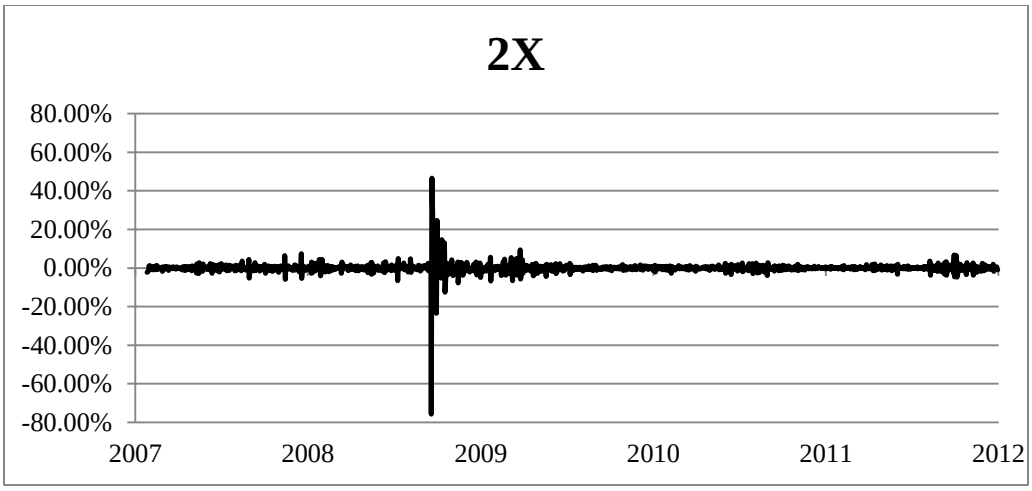


Figure 9: -1x S&P 600 Tracking Fund's Daily Excess Returns

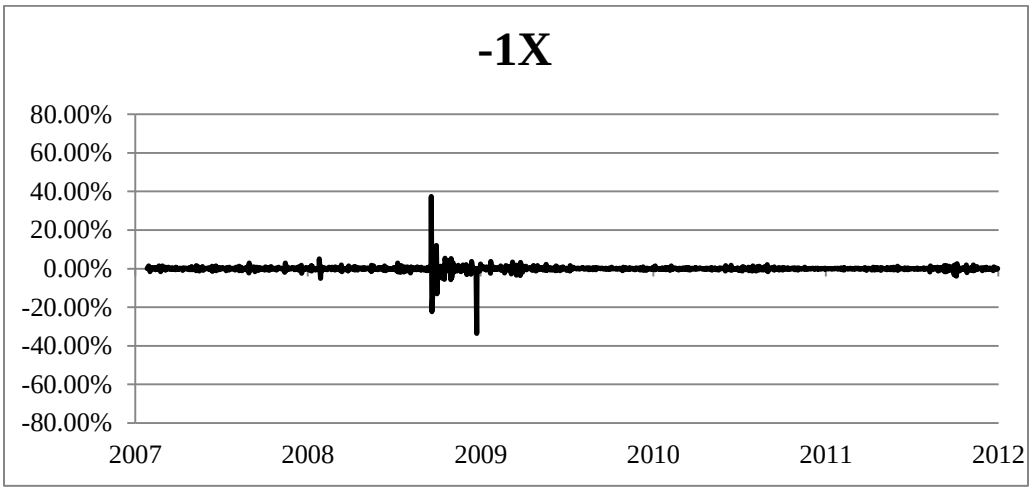
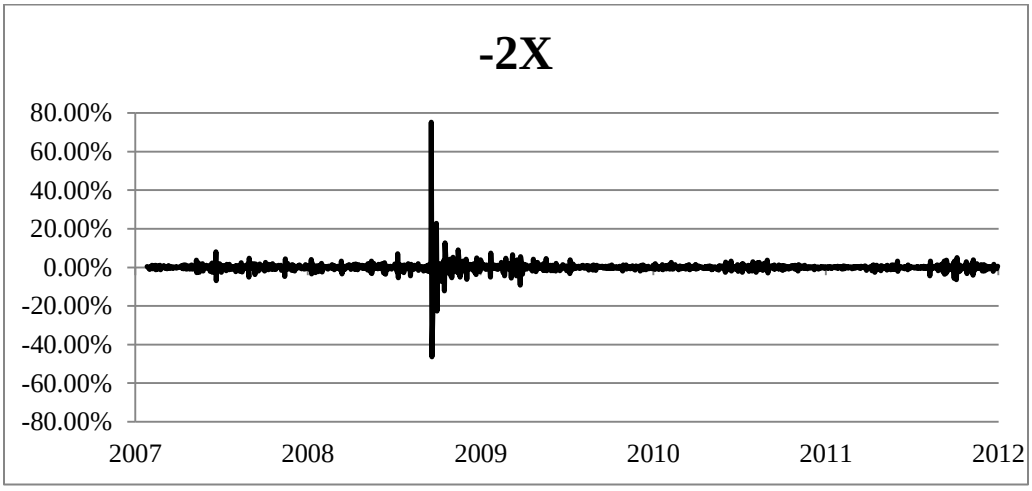


Figure 10: -2x S&P 600 Tracking Funds Daily Excess Returns



The S&P 600 funds produce large deviations from the expected mean of zero. Having large deviations in the data does not allow for the model to produce a good fit.

All of the funds seem to have relatively large deviations around the same time frame. Upon further analysis, it appears that the largest deviations for the funds (excluding the 3x S&P 500 tracking ETF: UPRO) occur during October of 2008. This month was the when the financial crisis caused the market to crash. In particular, October 13th is when most of the funds have their largest deviation. On that day, the S&P 500 surged 104 points, over eleven percent, as investors flooded money back into the market assuming the worst was over.

The tracking error of each ETF is calculated as the standard deviation of the fund's excess returns over time. Looking at Table 5, above, the 2x, 3x, -1x, and -2x S&P 500 tracking funds had tracking errors of 0.4258%, 0.3398%, 0.2428%, and 0.3926% respectably, over the five years. In contract, during the same period, the S&P 400 funds, MVV, MYY, and MZZ had tracking errors of 0.6018%, 0.5771%, and 0.5602% respectably. Finally, the 2x, -1x, and -2x S&P 600 funds had tracking errors of 3.1458%, 1.8538%, and 3.1050%, respectably over the time period specified. From this, I conclude that the S&P 400 and S&P 500 tracking funds track relatively well compared to the S&P 600 tracking funds.

Weekly Returns

To examine tracking ability with longer holding periods, I also regressed the fund's weekly returns over the appropriate multiplied indices. The results for weekly returns of the S&P 500 tracking funds are shown in table 8 below.

Table 8: Weekly S&P 500 Fund Regressions

Multiple	No. of Observations	Alpha (P-Value)*	Beta (P-Value)*	Adjusted R-Squared
2X	261	-0.0008	0.9752	0.9938
		0.0096	0.0000	
3X	131	-0.0004	1.0070	0.9971
		0.2564	0.1468	
-1X	261	0.0001	0.9782	0.9904
		0.6531	0.0003	
-2X	261	-0.0003	0.9858	0.9856
		0.6035	0.0560	

*P-Values for alphas are for testing against an alpha of zero. P-Values for betas are for testing against a beta of one.

The 2x fund has a p-value of .0096. This is less than .05, so I can reject the null that the alpha is zero. The p-values for the 3x, -1x, and -2x are all larger than .05, so I cannot reject that the alphas are zero for these funds. The p-values for the betas are .0000, .1468, .0003, and .0560 for the 2x, 3x, -1x, and -2x funds respectably. This allows me to reject the null that the betas are one for the 2x and -1x funds, but not for the 3x and -2x funds. The adjusted R-squareds for these models are 99.38%, 99.71%, 99.04%, and 98.56% for SSO, UPRO, SH, and SDS respectably. This shows that the models are still a good fit for the data using weekly returns. The funds are tracking well.

The results for the S&P 400 funds are in table 9 below.

Table 9: Weekly S&P 400 Fund Regressions

Multiple	No. of Observations	Alpha (P-Value)*	Beta (P-Value)*	Adjusted R-Squared
2X	261	-0.0007	0.9733	0.9928
		0.1043	0.0000	
-1X	261	-0.0005	0.9604	0.9268
		0.4130	0.0186	
-2X	261	-0.0003	0.9807	0.9721
		0.7117	0.0619	

*P-Values for alphas are for testing against an alpha of zero. P-Values for betas are for testing against a beta of one.

The p-values of the alphas are all greater than .05, so I cannot reject the null that the alphas are zero. Looking at the p-values for the betas- .0000, .0186, and .0619 for the 2x, -1x, and -2x funds respectably- I can reject that the betas are one for the 2x and -1x funds, but not for the -2x fund at the .05 significance level. The adjusted R-squareds for the funds are 99.28%, 92.68%, and 97.21% for MVV, MYY, and MZZ respectably. This shows that the models are still a good fit for the weekly data, and the funds are tracking considerably well.

The S&P 600 tracking funds results are in table 10 below.

Table 10: Weekly S&P 600 Fund Regressions

Multiple	No. of Observations	Alpha (P-Value)*	Beta (P-Value)*	Adjusted R-Squared
2X	257	-0.0011	0.9026	0.9272
		0.4073	0.0000	
-1X	257	-0.0013	0.9101	0.7132
		0.3797	0.0132	
-2X	257	-0.0005	0.9145	0.9336
		0.7191	0.0000	

*P-Values for alphas are for testing against an alpha of zero. P-Values for betas are for testing against a beta of one.

Again, looking at the p-values for the alphas, I cannot reject the null that the alphas are zero for any of the funds. The p-values for the betas, however, are all less than .05, allowing me to reject that the betas are one for all of the funds. The adjusted R-squareds for the models are 92.72%, 71.32%, and 93.36%, for SAA, SBB, and SDD respectably. This tells us the models for SAA and SDD fit the data fairly well, while the model for SBB does not fit very well. In table 11 below, I compare the excess returns and tracking errors of the funds.

Table 11: Weekly Fund Excess Returns

Fund	Observation	Mean	Std. Dev.	Min	Max
S&P 500 Funds					
2X	261	-0.0852%	0.5364%	-4.0880%	5.5480%
3X	131	-0.0371%	0.4371%	-3.1110%	1.2190%
-1X	261	0.0100%	0.3274%	-2.2620%	2.1600%
-2X	261	-0.0242%	0.7953%	-5.8460%	4.7040%
S&P 400 Funds					
2X	261	-0.0723%	0.6774%	-5.0770%	4.6770%
-1X	261	-0.0484%	1.0629%	-13.9490%	4.4040%
-2X	261	-0.0245%	1.3025%	-8.3430%	9.7970%
S&P 600 Funds					
2X	257	-0.1387%	2.2938%	-12.0900%	10.9510%
-1X	257	-0.1208%	2.4689%	-35.7660%	6.0760%
-2X	257	-0.0223%	2.1863%	-12.1320%	10.2970%

With a range of -5.846 to 5.548%, the S&P 500 funds have a small range of excess returns compared to the -13.949% to 9.797% range for the S&P 400 funds and the -35.766% to 10.951% range for the S&P 600 tracking funds. This is shown visually in Figures 11-20 below.

Figure 11: 2x S&P 500 Tracking Fund's Weekly Excess Returns

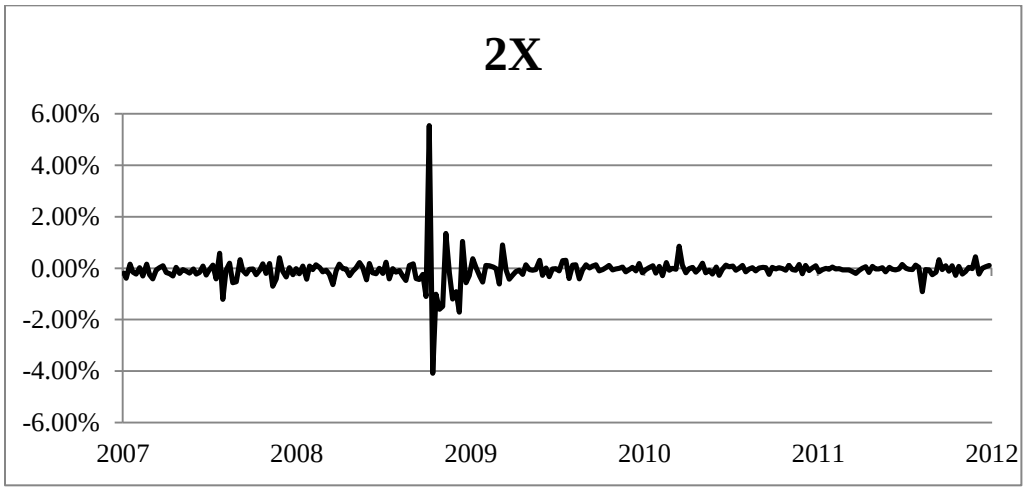


Figure 12: 3x S&P 500 Tracking Fund's Weekly Excess Returns

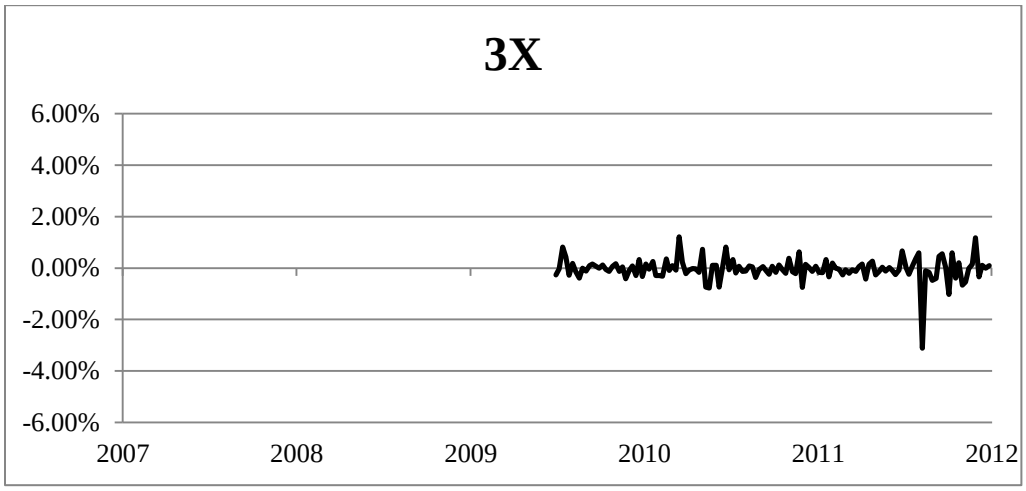


Figure 13: -1x S&P 500 Tracking Weekly Excess Returns

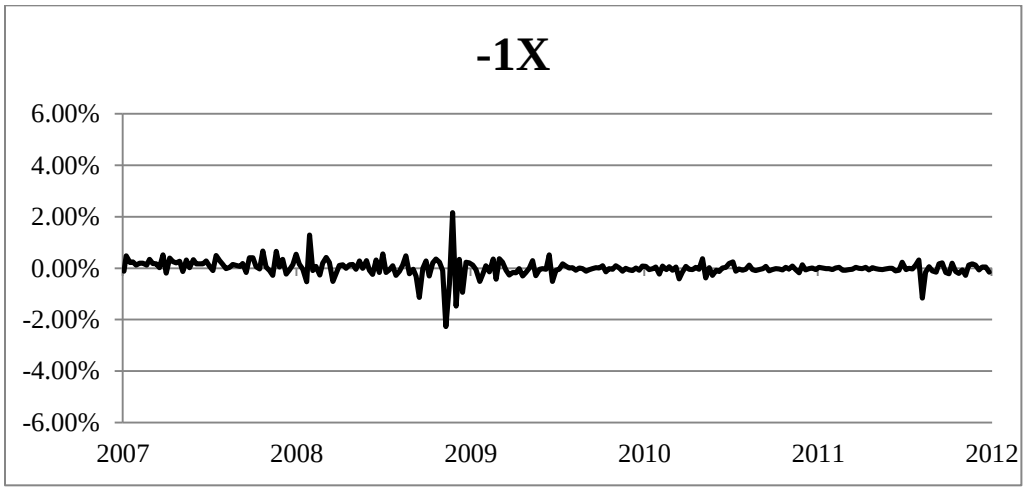


Figure 14: -2x S&P 500 Tracking Weekly Excess Returns

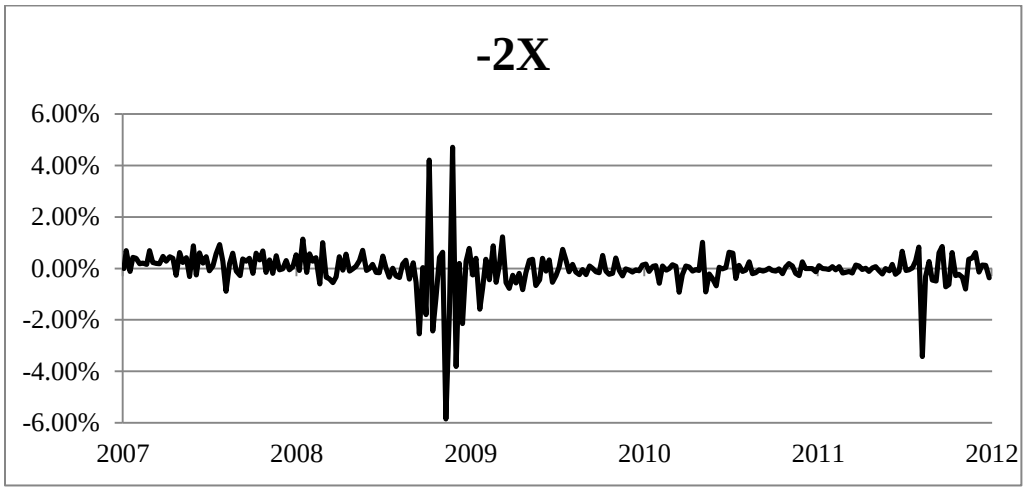


Figure 15: 2x S&P 400 Tracking Fund's Weekly Excess Returns

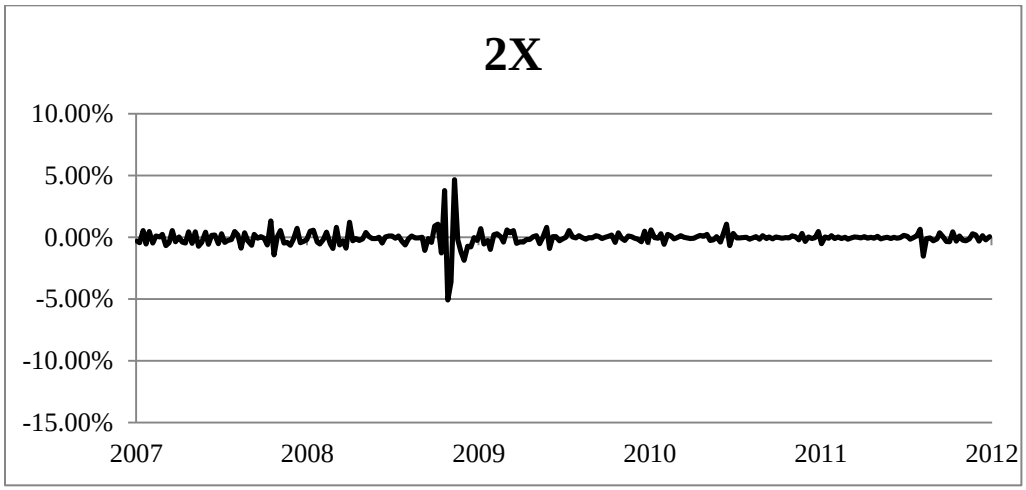


Figure 16: -1x S&P 400 Tracking Fund's Weekly Excess Returns

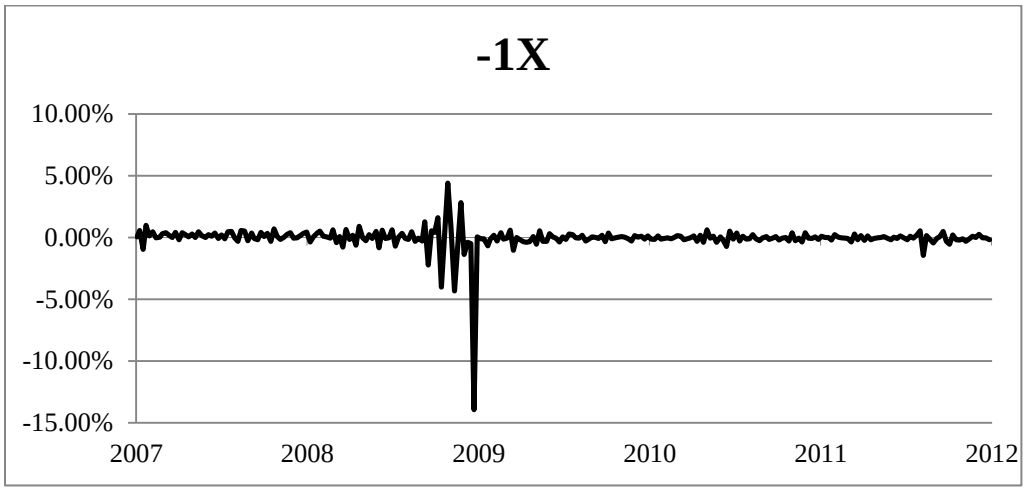


Figure 17: -2x S&P 400 Tracking Fund's Weekly Excess Returns

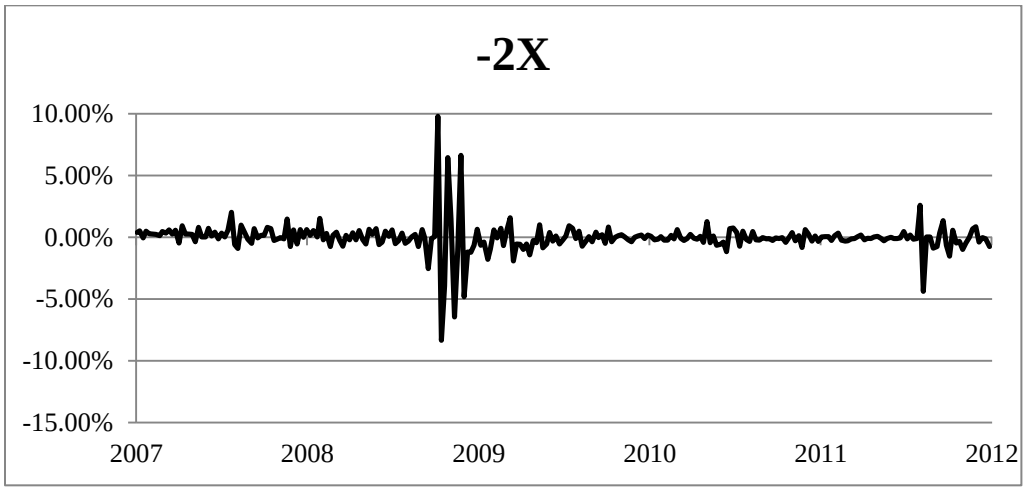


Figure 18: 2x S&P 600 Tracking Fund's Weekly Excess Returns

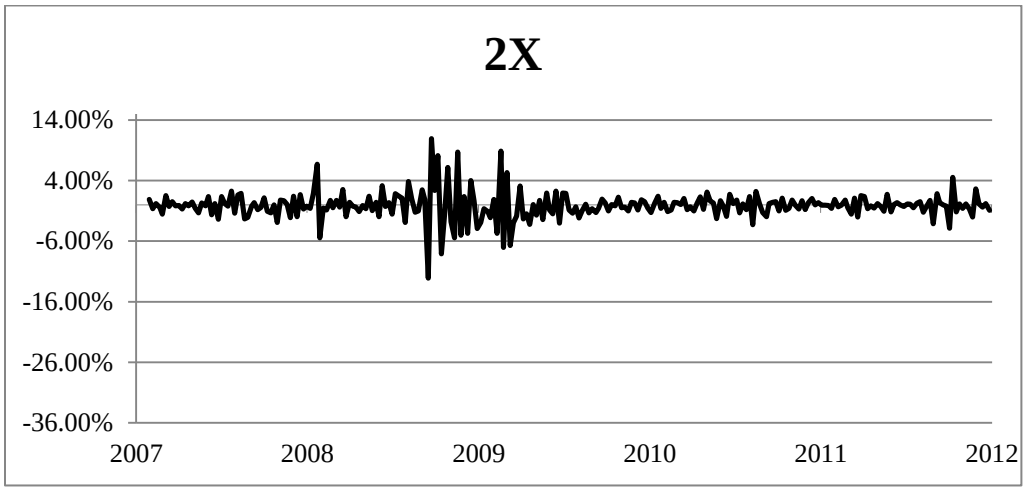


Figure 19: -1x S&P 600 Tracking Fund's Weekly Excess Returns

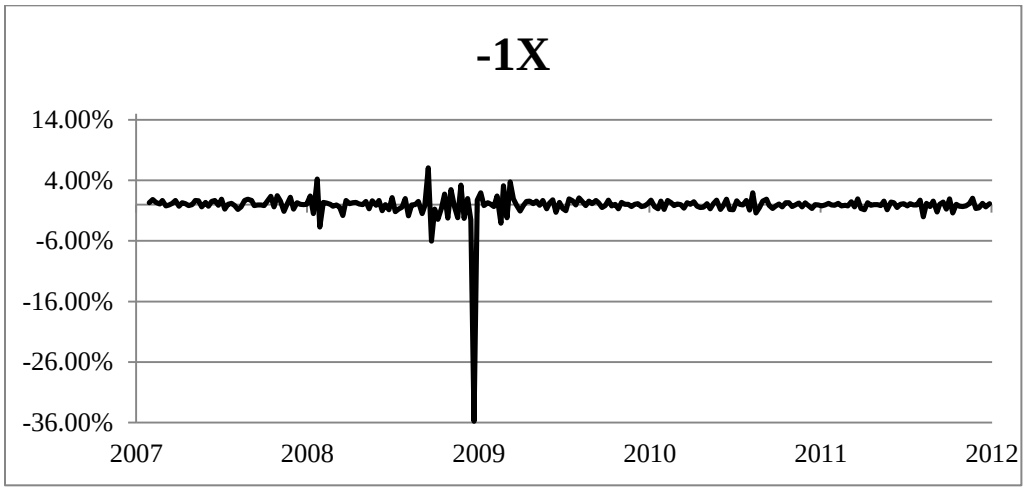
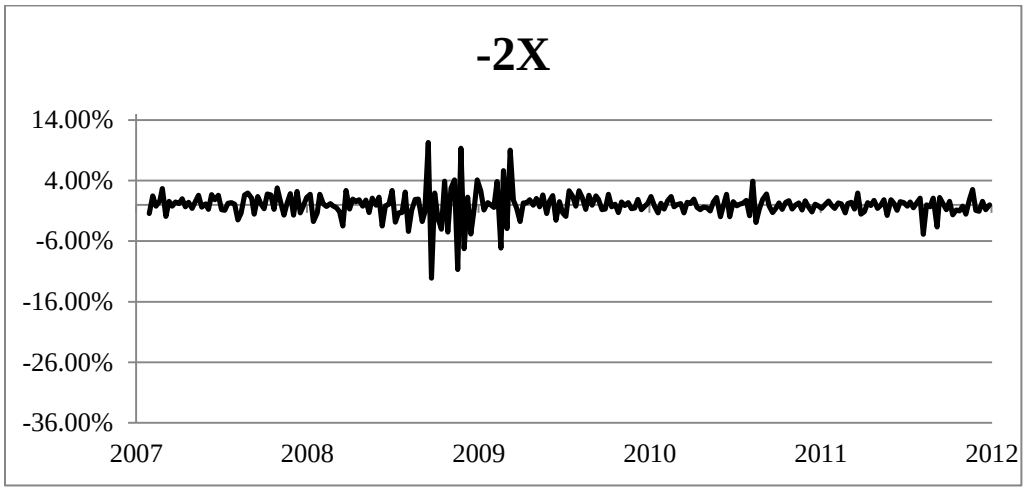


Figure 20: -2x S&P 600 Tracking Fund's Weekly Excess Returns



The weekly tracking errors of the S&P 500 tracking funds, SSO, UPRO, SH, and SDS are 0.5364%, 0.4371%, 0.3274%, and 0.7953% respectably, over the five years analyzed. The S&P 400 tracking funds, MVV, MYY, and MZZ, had weekly tracking error of 0.6774%, 1.0629%, and 1.3025% respectably. Finally, the S&P 600 funds, SAA, SBB, and SDD had weekly tracking errors of 2.2938%, 2.4689%, and 2.1863% respectably, over the time period

specified. By comparison, the S&P 600 tracking funds did not track as well as the S&P 500 and S&P 400 tracking funds.

Monthly Returns

Finally, to look at an even longer holding period, I regress the funds' monthly returns on the appropriate multiples of the index returns. The results for the S&P 500 tracking funds are shown in table 12 below.

Table 12: Monthly S&P 500 Fund Regressions

Multiple	No. of Observations	Alpha (P-Value)*	Beta (P-Value)*	Adjusted R-Squared
2X	60	-0.0048 0.0000	1.0209 0.0178	0.9958
3X	30	-0.0052 0.0078	1.0401 0.0033	0.9958
-1X	60	-0.0015 0.1289	0.9495 0.0079	0.9784
-2X	60	-0.0075 0.0098	0.8885 0.0001	0.9517

*P-Values for alphas are for testing against an alpha of zero. P-Values for betas are for testing against a beta of one.

Using p-values again, I can reject that the alphas are zero for the 2x, 3x, and -2x funds at the .05 significance level; however, I cannot for the -1x fund. For the 2x, 3x, and -2x, I find that the alphas are negative. The p-values for the betas are all less than .05, allowing me to reject that the betas are one for all of the funds. The adjusted R-squareds for these models are 99.58%, 99.58%, 97.84%, and 95.17% for SSO, UPRO, SH, and SDS respectably. This shows that the models are still a good fit for the data even at the monthly range. The funds are still tracking very well.

The results for the S&P 400 funds are in table 13 below.

Table 13: Monthly S&P 400 Fund Regressions

Multiple	No. of Observations	Alpha (P-Value)*	Beta (P-Value)*	Adjusted R-Squared
2X	60	-0.0046	1.0117	0.9962
		0.0000	0.1544	
-1X	60	-0.0044	0.9794	0.8881
		0.1364	0.6501	
-2X	60	-0.0098	0.9099	0.9627
		0.0019	0.0003	

*P-Values for alphas are for testing against an alpha of zero. P-Values for betas are for testing against a beta of one.

For the 2x and -2x funds, the p-values for the alphas are less than .05, allowing me to reject the null that the alphas are zero. The p-value for the -1x fund is larger than .05, so I cannot reject the null that the alpha is zero for this fund. Looking at the p-values for the betas, the 2x and -1x fund have values larger than .05, not allowing me to reject that the betas are one. The -2x fund, however, has a p-value of .0019, which is less than .05, allowing me to reject that the beta is one. The adjusted R-squareds for the funds are 99.62%, 88.81%, and 96.27% for MVV, MYY, and MZZ respectably. This shows that the models are still a good fit for the monthly data, and the funds are tracking considerably well.

The S&P 600 tracking funds regression results are in table 14 below.

Table 14: Monthly S&P 600 Fund Regressions

Multiple	No. of Observations	Alpha (P-Value)*	Beta (P-Value)*	Adjusted R-Squared
2X	59	-0.0068	0.9608	0.9729
		0.0269	0.0677	
-1X	59	-0.0077	0.9595	0.6631
		0.2312	0.6522	
-2X	59	-0.0097	0.8751	0.942
		0.0201	0.0001	

*P-Values for alphas are for testing against an alpha of zero. P-Values for betas are for testing against a beta of one.

The p-values for the 2x and -2x funds' alphas are both less than .05, allowing me to reject the null that the alphas are zero for these funds. The p-value for the -1x fund is larger than .05, not allowing me to reject the null hypothesis that the alpha is zero. The p-values for the betas of 2x and -1x are both larger than .05, not allowing me to reject that the betas are one, however, the p-value for the beta of the -2x fund is less than .05, and I can therefore reject the null that the beta is one. The adjusted R-squareds for the models are 97.29%, 66.31%, and 94.20%, for SAA, SBB, and SDD respectably. This tells us that the models for SAA and SDD again fit the data considerably well, while the model for SBB still does not fit very well.

In table 15 below, I compare the excess return and tracking error of the funds.

Table 15: Monthly Excess Returns

Fund	Observation	Mean	Std. Dev.	Min	Max
S&P 500 Funds					
2X	60	-0.4756%	0.7449%	-3.0620%	1.0420%
3X	30	-0.3664%	1.1106%	-4.6020%	1.9450%
-1X	60	-0.1462%	0.8072%	-3.1590%	1.3340%
-2X	60	-0.7229%	2.4696%	-13.2750%	1.6410%
S&P 400 Funds					
2X	60	-0.4530%	0.8105%	-3.8010%	0.8800%
-1X	60	-0.4300%	2.2299%	-15.4660%	1.9670%
-2X	60	-0.8962%	2.5753%	-12.0050%	2.3890%
S&P 600 Funds					
2X	59	-0.7190%	2.3549%	-6.5610%	5.3220%
-1X	59	-0.7553%	4.8679%	-35.9980%	3.1910%
-2X	59	-0.8614%	3.5827%	-13.6640%	6.8440%

With a range of -13.275% to 1.945%, the S&P 500 funds have a smaller range of excess returns compared to the -15.466% to 2.389% range for the S&P 400 funds and the -35.998% to 6.844% range for the S&P 600 tracking funds. I also find that all of the mean excess returns are negative for a monthly holding period. This is shown in Figures 21-30 below.

Figure 21: 2x S&P 500 Tracking Fund's Monthly Excess Returns

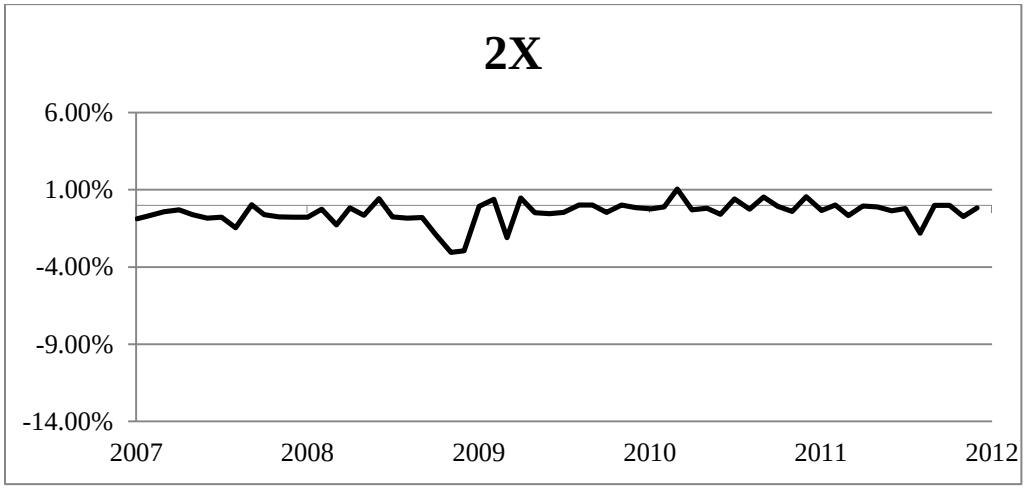


Figure 22: 3x S&P 500 Tracking Fund's Monthly Excess Returns

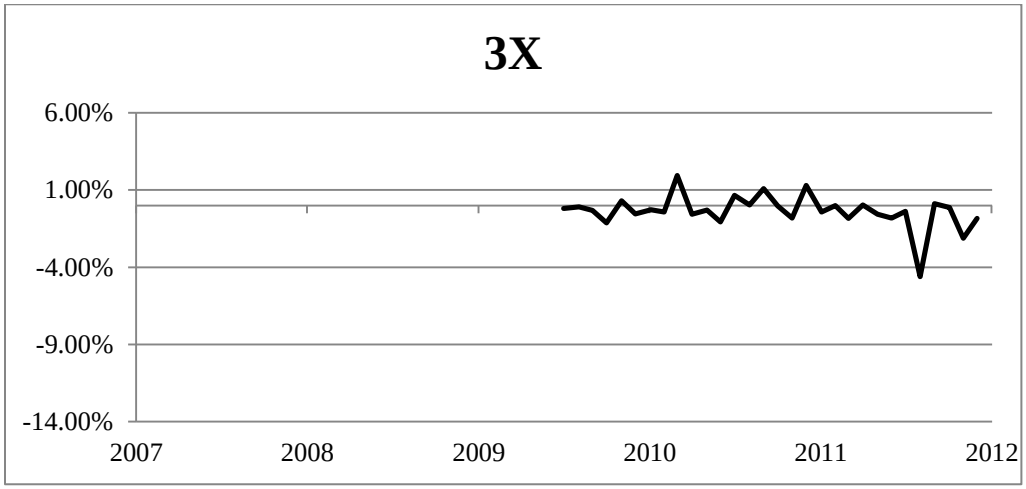


Figure 23: -1x S&P 500 Tracking Fund's Monthly Excess Returns

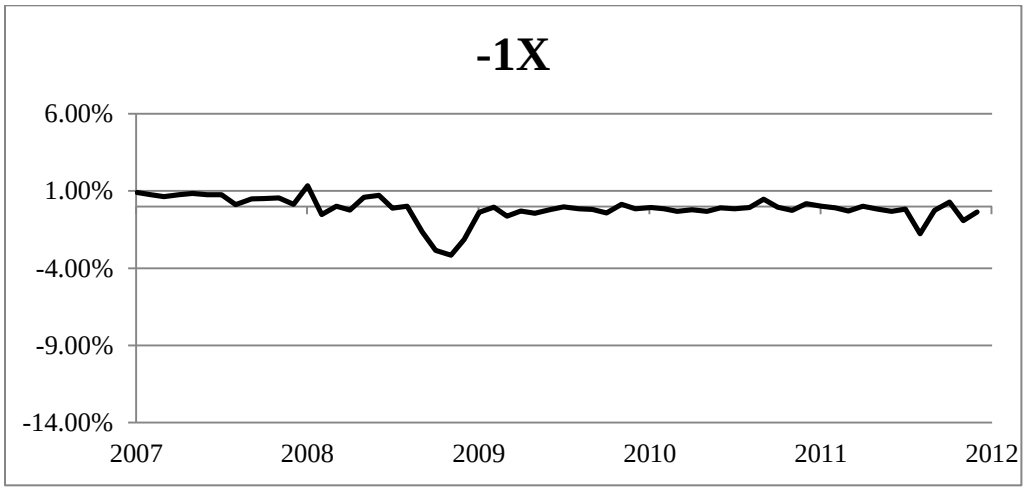


Figure 24: -2x S&P 500 Tracking Fund's Monthly Excess Returns

Figure 25: 2x S&P 400 Tracking Fund's Monthly Excess Returns

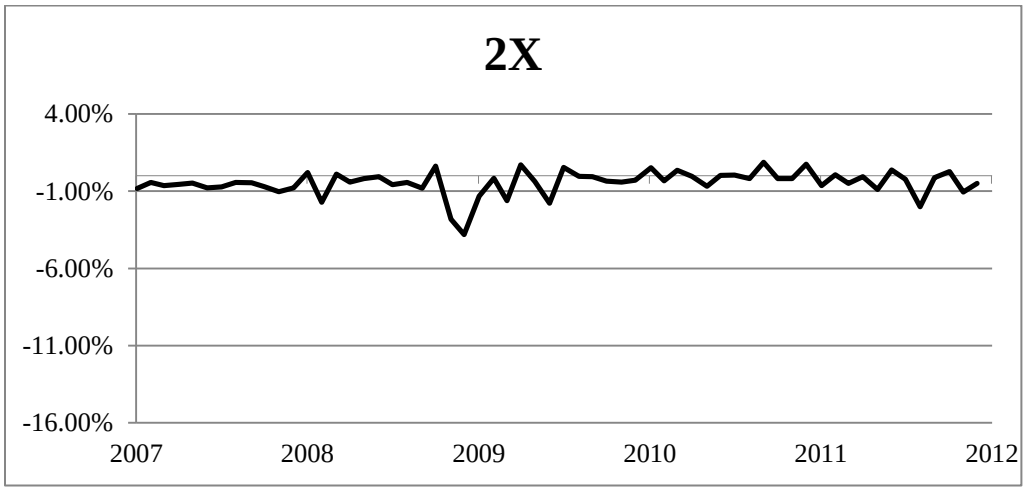


Figure 26: -1x S&P 400 Tracking Fund's Monthly Excess Returns

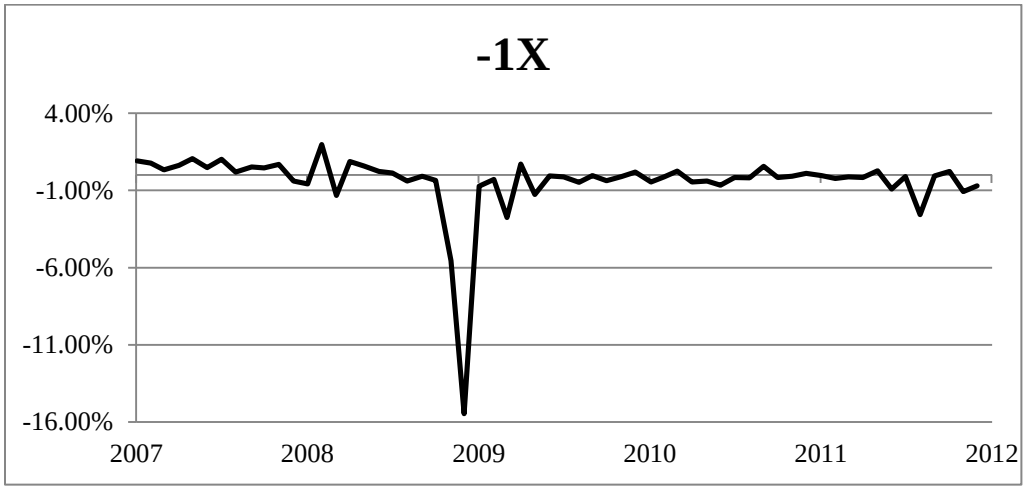


Figure 27: -2x S&P 400 Tracking Fund's Monthly Excess Returns

Figure 28: 2x S&P 600 Tracking Fund's Monthly Excess Returns

Figure 29: -1x S&P 600 Tracking Fund's Monthly Excess Returns

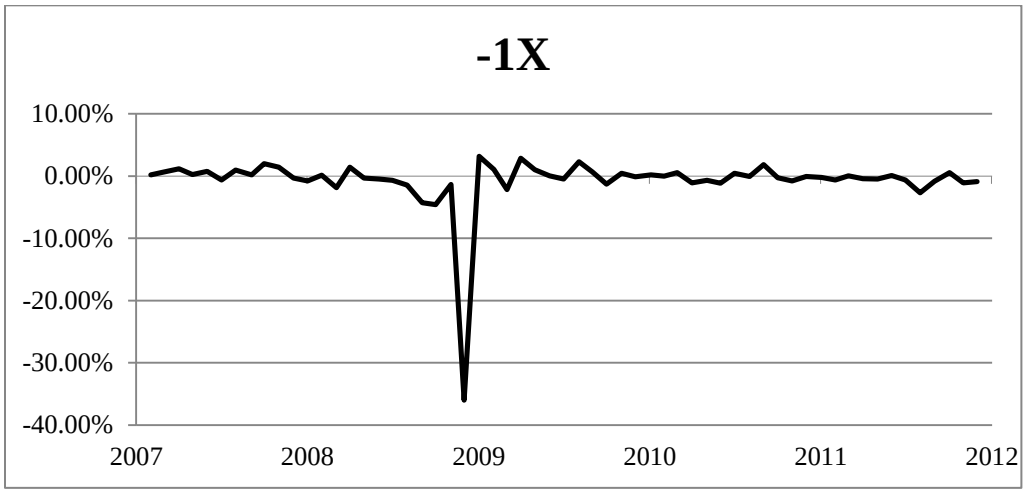
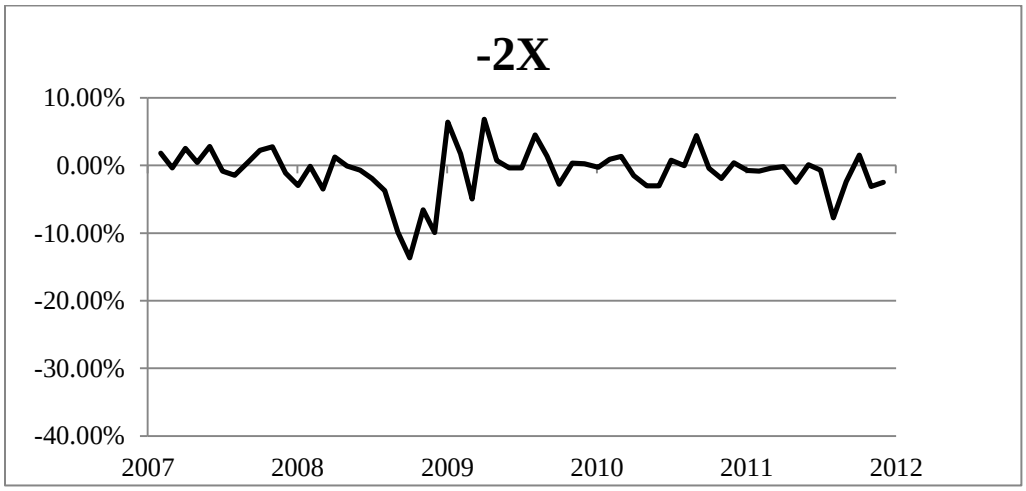


Figure 30: -2x S&P 600 Tracking Fund's Monthly Excess Returns



The monthly tracking errors of the 2x, 3x, -1x, and -2x S&P 500 tracking funds are 0.7449%, 1.1106%, 0.82072%, and 2.4696% respectably, over the five years analyzed. The 2x, -1x, and -2x S&P 400 tracking funds, had monthly tracking error of 0.8105%, 2.2299%, and 2.5753% respectably. Finally, the monthly tracking error of the 2x, -1x, and -2x S&P 600 tracking funds, are 2.3549%, 4.8679%, and 3.5827% respectably, over the time period specified.

As for shorter time horizons, the S&P 600 tracking funds did not track as well as the S&P 500 and S&P 400 tracking funds.

To analyze the distribution of the excess returns, I plot them on histograms. The daily results are seen in figures 31-40 below.

Figure 31: 2x S&P 500 Tracking Fund's Daily Excess Return Distribution

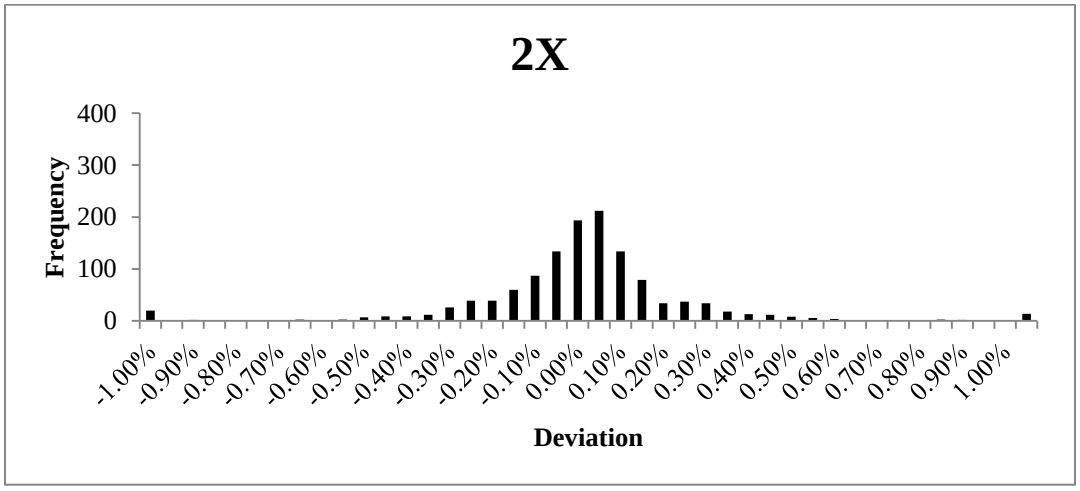


Figure 32: 3x S&P 500 Tracking Fund's Daily Excess Return Distribution

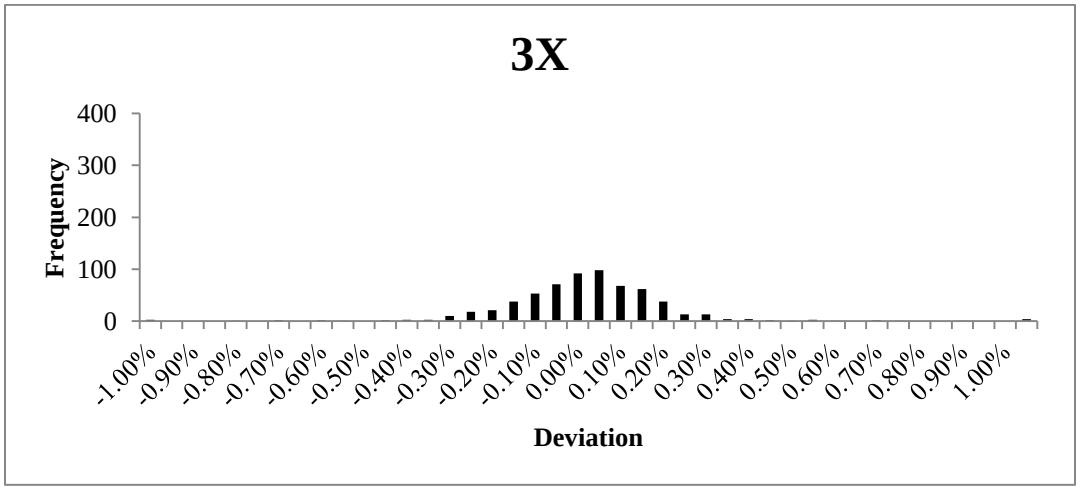


Figure 33: -1x S&P 500 Tracking Fund's Daily Excess Return Distribution

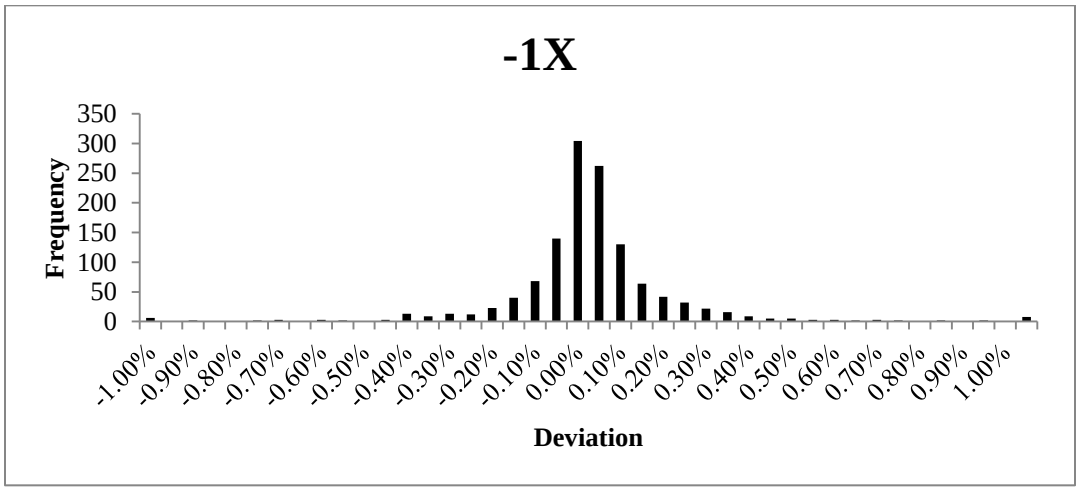


Figure 34: -2x S&P 500 Tracking Fund's Daily Excess Return Distribution

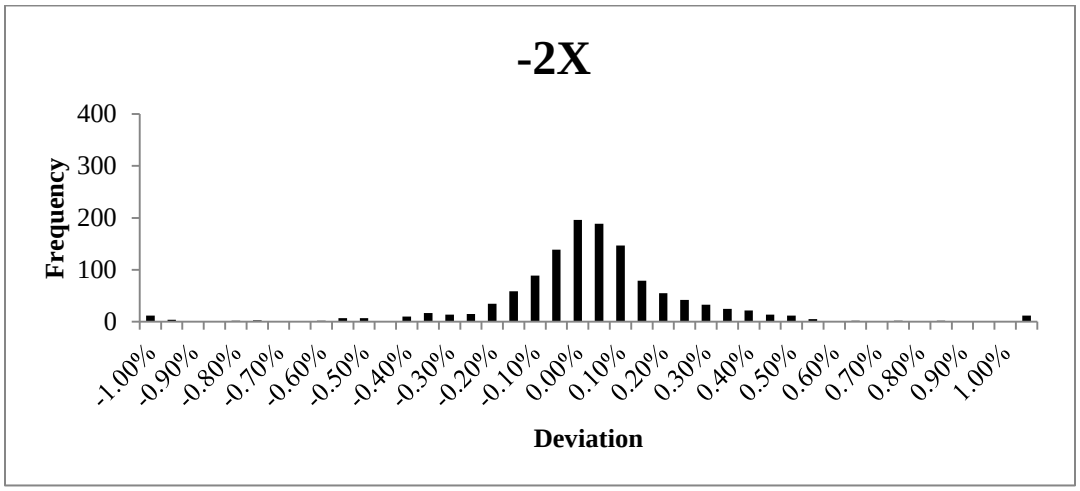


Figure 35: 2x S&P 400 Tracking Fund's Daily Excess Return Distribution

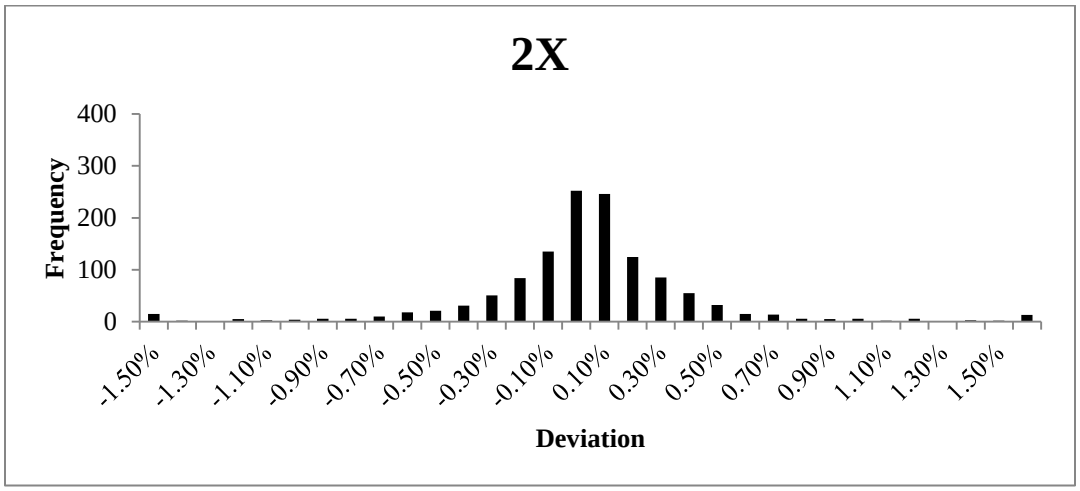


Figure 36: -1x S&P 400 Tracking Fund's Daily Excess Return Distribution

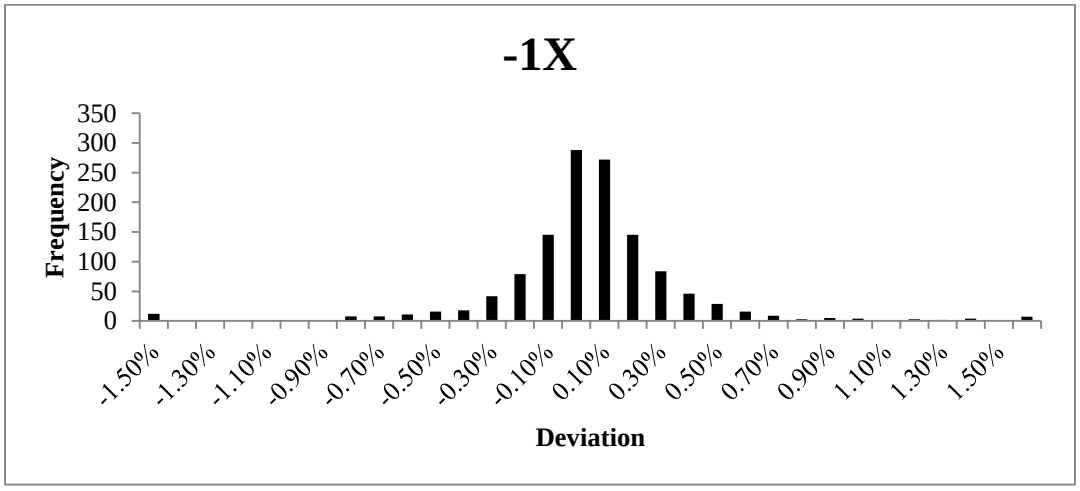


Figure 37: -2x S&P 400 Tracking Fund's Daily Excess Return Distribution

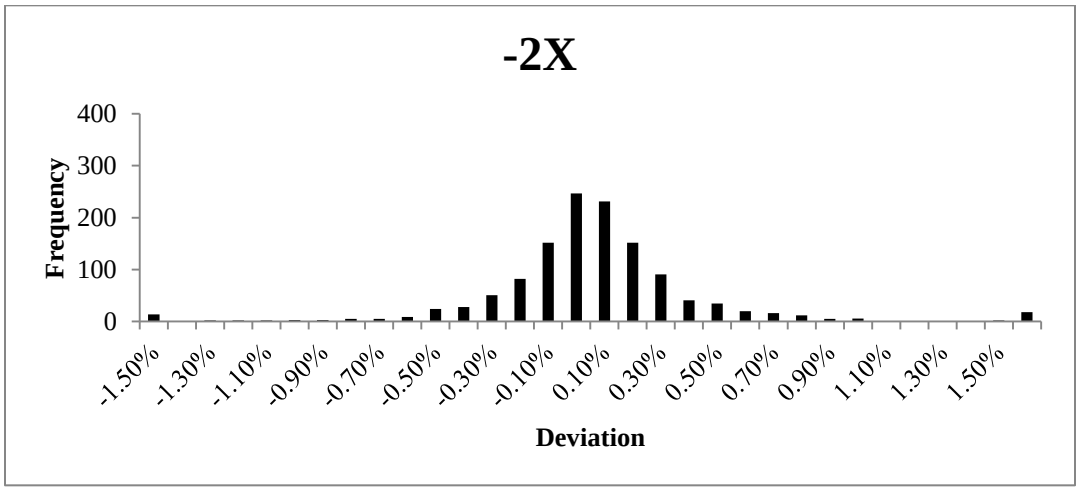


Figure 38: 2x S&P 600 Tracking Fund's Daily Excess Return Distribution

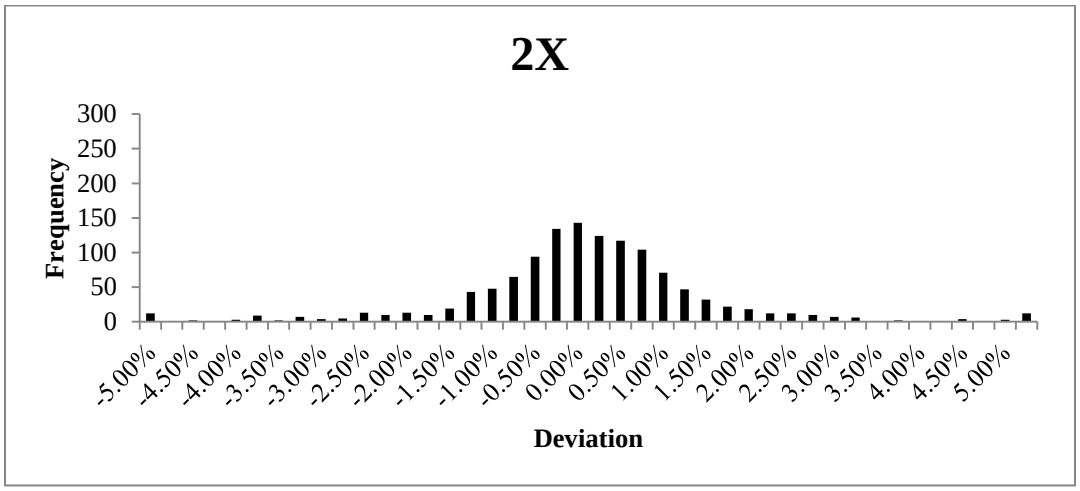


Figure 39: -1x S&P 600 Tracking Fund's Daily Excess Return Distribution

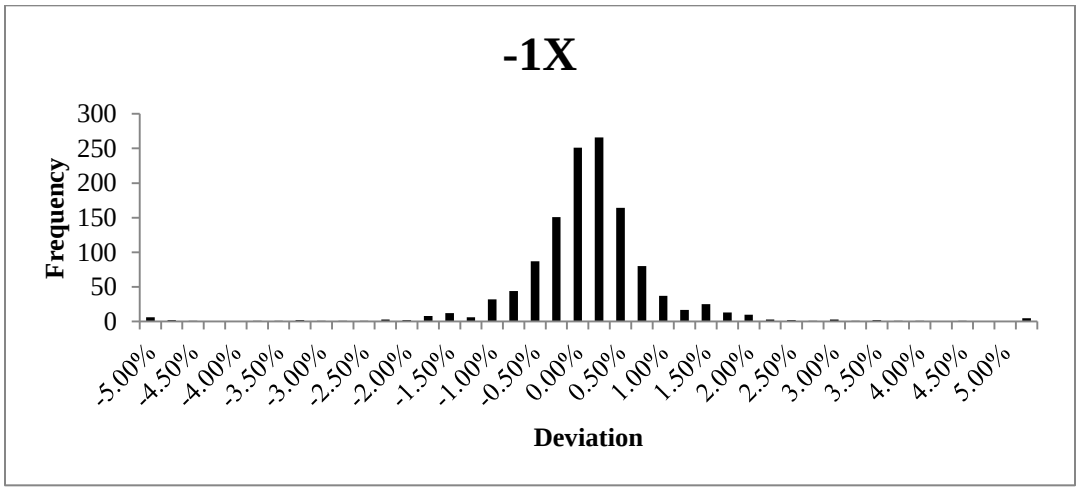
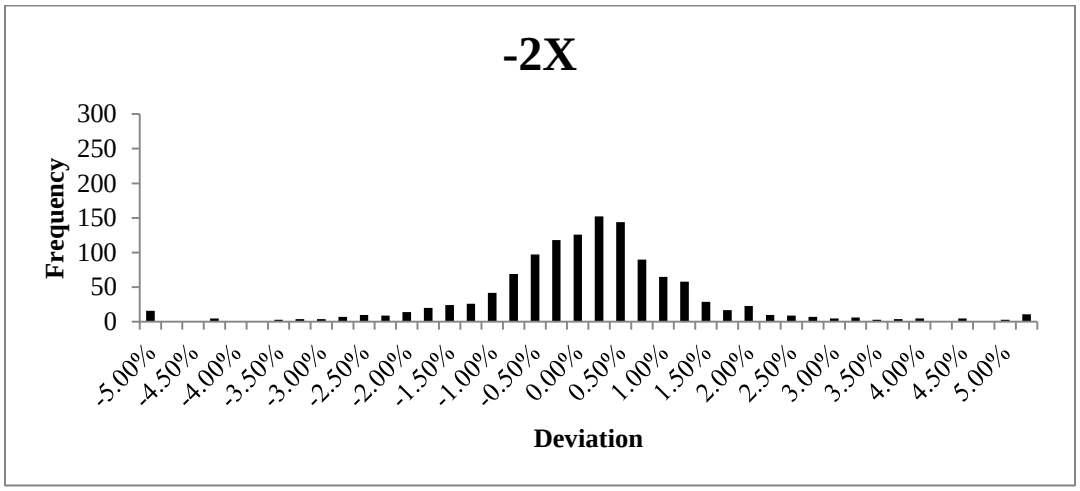


Figure 40: -2x S&P 600 Tracking Fund's Daily Excess Return Distribution



The distribution of returns seems to crowd between negative half a percent and half a percent for the S&P 500 and S&P 400 funds, with a few points falling outside this area. The S&P 600 funds seem to have a wider distribution that crowd between negative one and a half percent and one and a half percent, with a few points falling outside of that. This shows that

there are a few outliers in the data. This also confirms our previous analysis that the S&P 500 and S&P 400 funds seem to track better than the S&P 600 funds, with smaller deviations.

Skewness in the distribution is important to determine where our returns are more likely to end up. The results for the daily returns are in table 16 below.

Table 16: Daily Excess Return Skewness

Daily Excess Return Skewness		
Fund	Skewness	P-Value
S&P 500		
2x	-2.6741	0.0000
3x	-0.609641	0.0000
-1x	2.7031	0.0000
-2x	3.1342	0.0000
S&P 400		
2x	-0.6493	0.0000
-1x	-12.3399	0.0000
-2x	0.0087	0.8987
S&P 600		
2x	-8.3942	0.0000
-1x	0.4068	0.0000
-2x	8.6847	0.0000

I find that the all funds have significant skewness except for the -2x S&P 400 tracking fund. I find that all bull funds have a negative skewness and all bear funds (with the exception of the previously stated fund) have a positive skewness. Negative skewness tells us that the mean excess return is less than the median excess return. This shows that investors are more likely to have an excess return that is larger than the average. The opposite is true for a positive skewness.

The results for the weekly returns are in figures 41-50 below.

Figure 41: 2x S&P 500 Tracking Fund's Weekly Excess Return Distribution

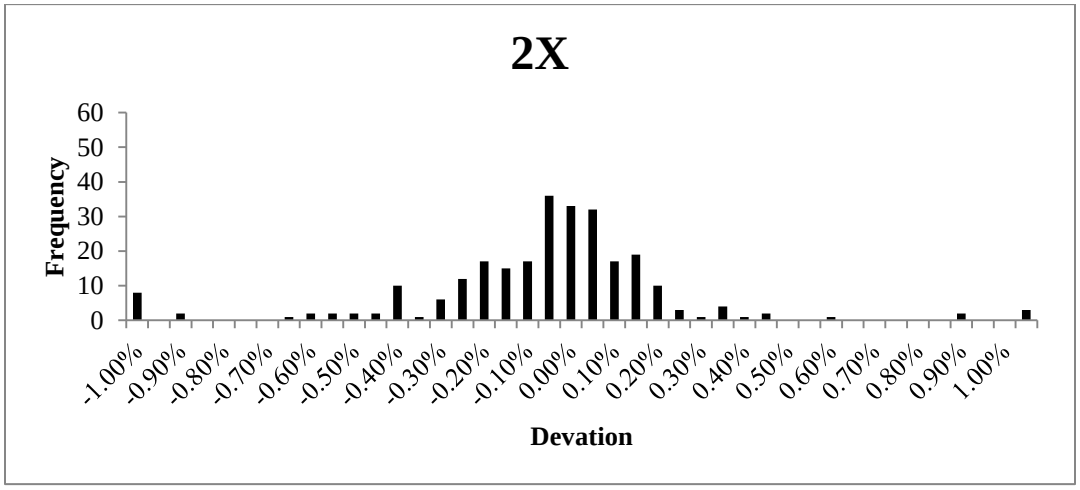


Figure 42: 3x S&P 500 Tracking Fund's Weekly Excess Return Distribution

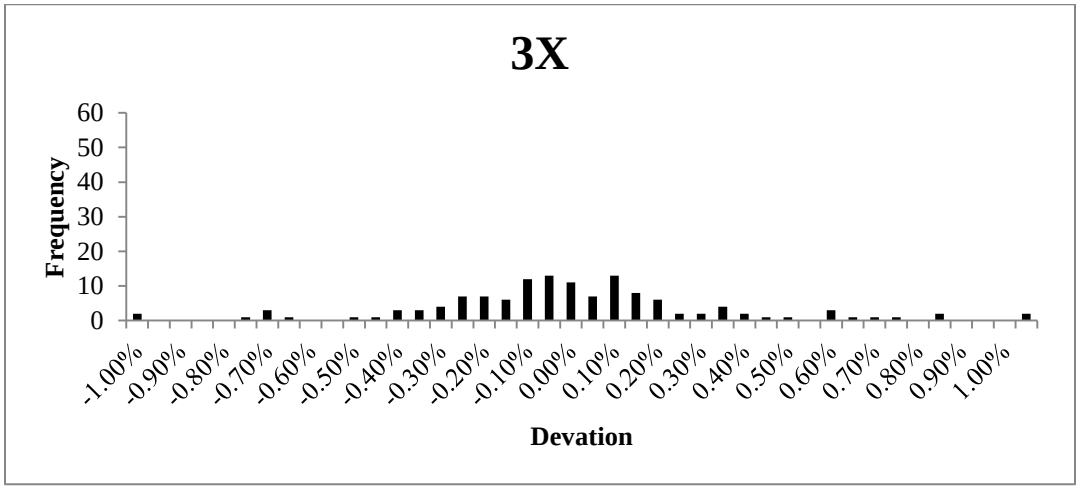


Figure 43: -1x S&P 500 Tracking Fund's Weekly Excess Return Distribution

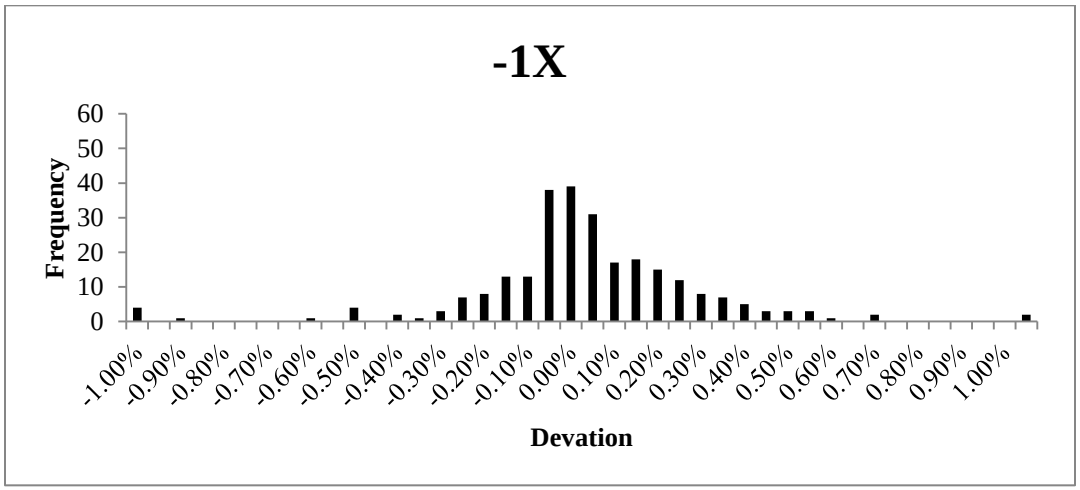


Figure 44: -2x S&P 500 Tracking Fund's Weekly Excess Return Distribution

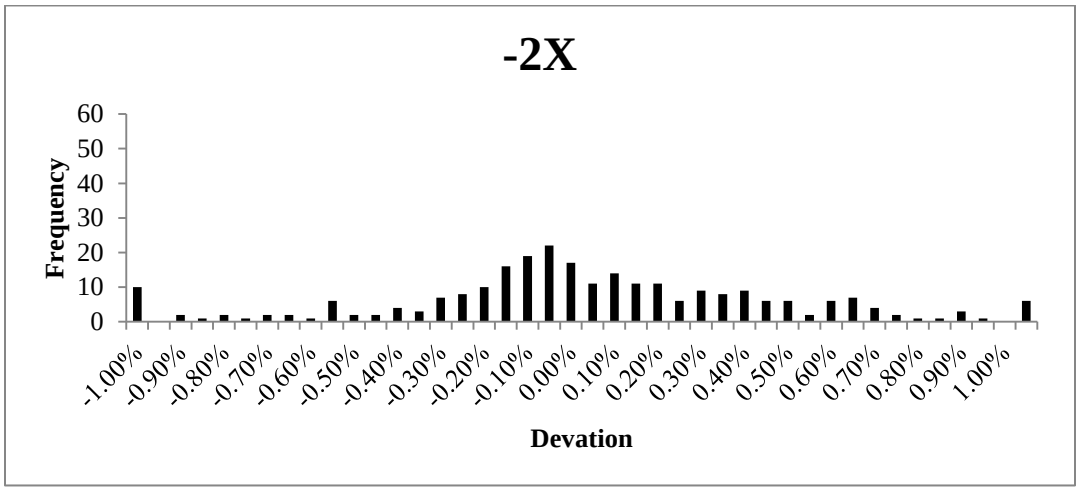


Figure 45: 2x S&P 400 Tracking Fund's Weekly Excess Return Distribution

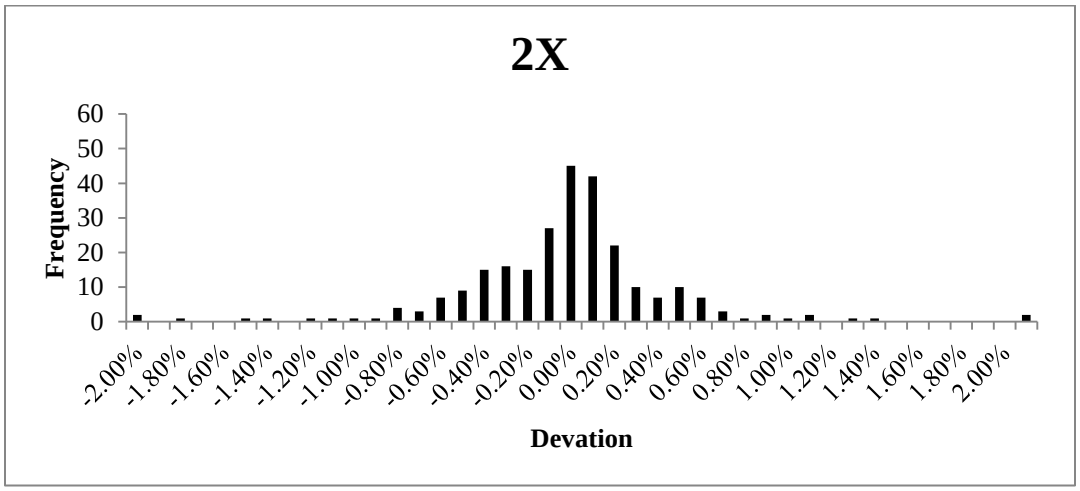


Figure 46: -1x S&P 400 Tracking Fund's Weekly Excess Return Distribution

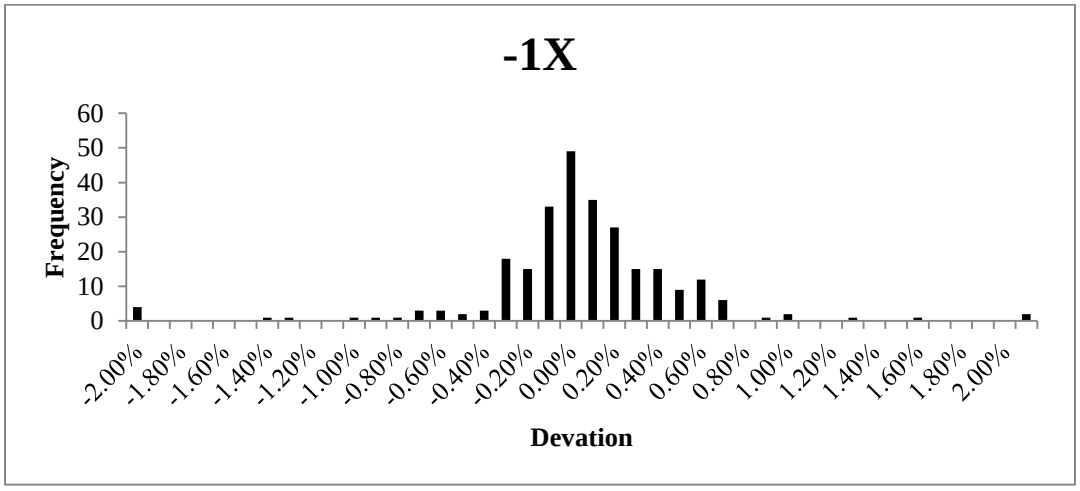


Figure 47: -2x S&P 400 Tracking Fund's Weekly Excess Return Distribution

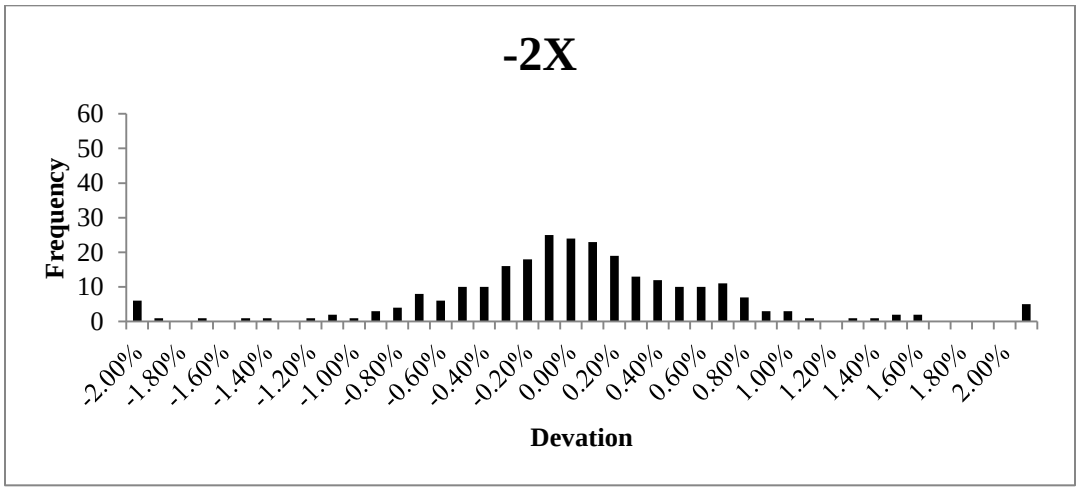


Figure 48: 2x S&P 600 Tracking Fund's Weekly Excess Return Distribution

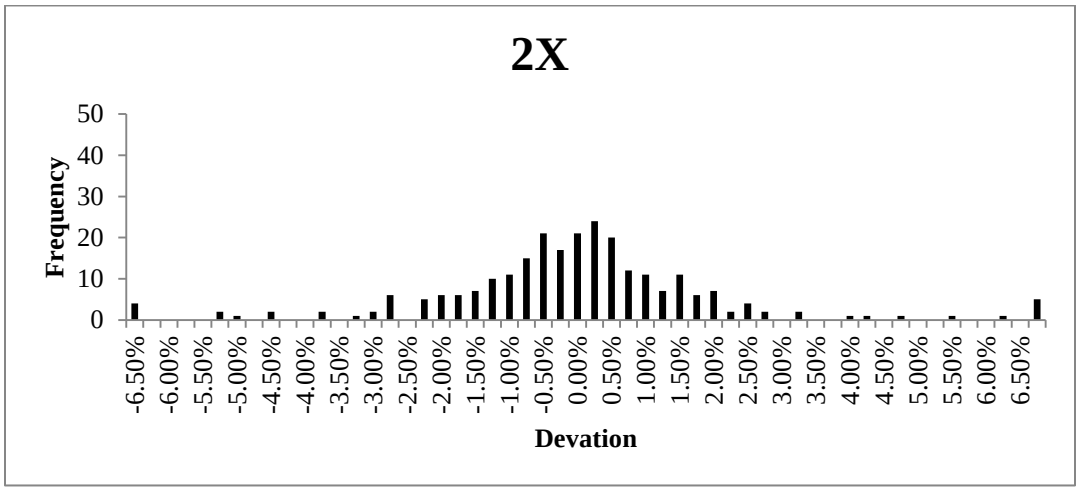


Figure 49: -1x S&P 600 Tracking Fund's Weekly Excess Return Distribution

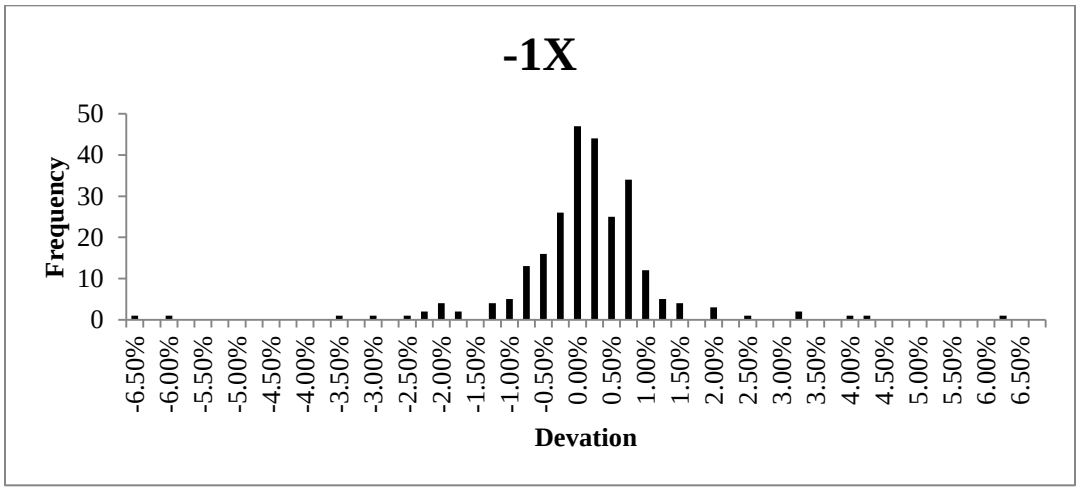
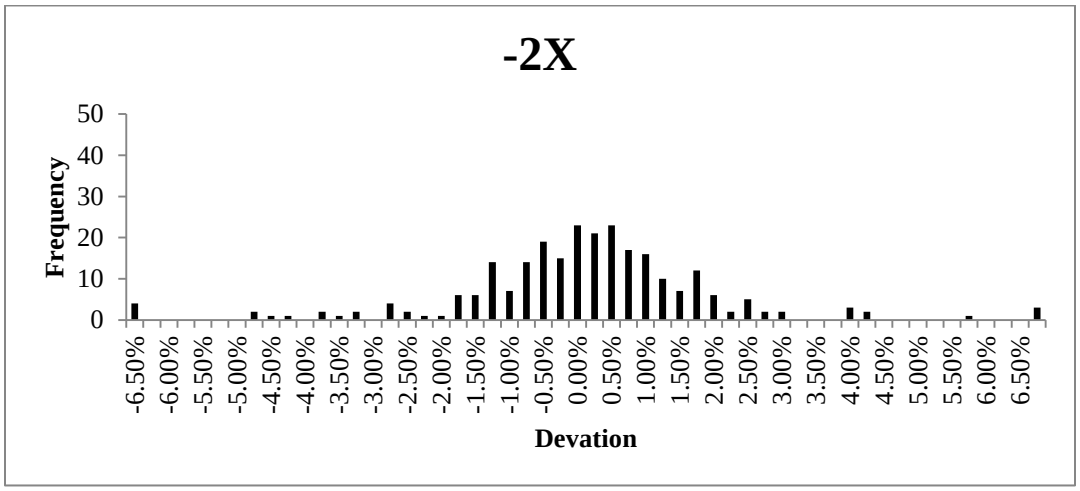


Figure 50: -2x S&P 600 Tracking Fund's Weekly Excess Return Distribution



The excess weekly returns seem to have a higher frequency between negative half a percent and half a percent for the S&P 500. The S&P 400 tracking funds seem to bunch between negative one and one percent. The S&P 600 tracking funds have a wider distribution crowding between negative two and a half percent and two and a half percent. All of the funds have outliers again, as seen by the range in the graphs.

Table 17: Weekly Excess Return Skewness

Weekly Excess Return Skewness		
Fund	Skewness	P-Value
S&P 500		
2x	2.6062	0.0000
3x	-2.3053	0.0000
-1x	-0.6646	0.0000
-2x	-1.1815	0.0000
S&P 400		
2x	-0.1388	0.3493
-1x	-8.7257	0.0000
-2x	0.7553	0.0000
S&P 600		
2x	0.2500	0.0973
-1x	-11.7356	0.0000
-2x	-0.4296	0.0055

For the weekly data, I find that all of the excess returns are skewed except for the 2x S&P 400 tracking fund and the 2x S&P 600 tracking fund. More funds have a negative skewness at the weekly holding period.

The monthly results are in figures 51-60 below.

Figure 51: 2x S&P 500 Tracking Fund's Monthly Excess Return Distribution

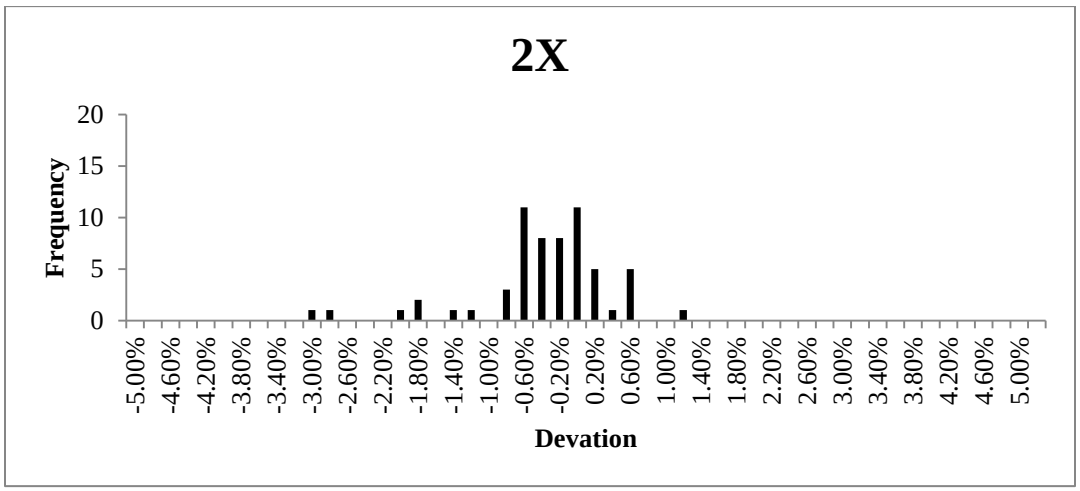


Figure 52: 3x S&P 500 Tracking Fund's Monthly Excess Return Distribution

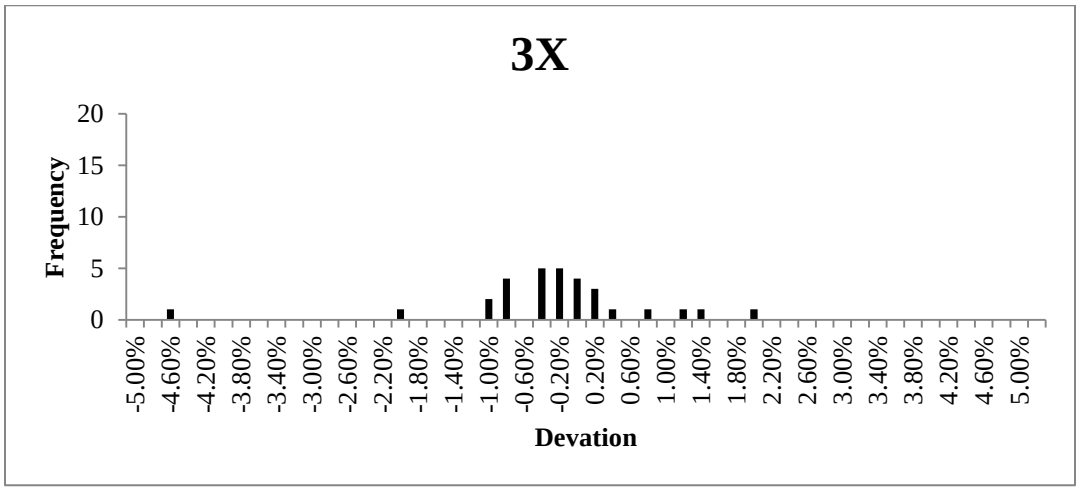


Figure 53: -1x S&P 500 Tracking Fund's Monthly Excess Return Distribution

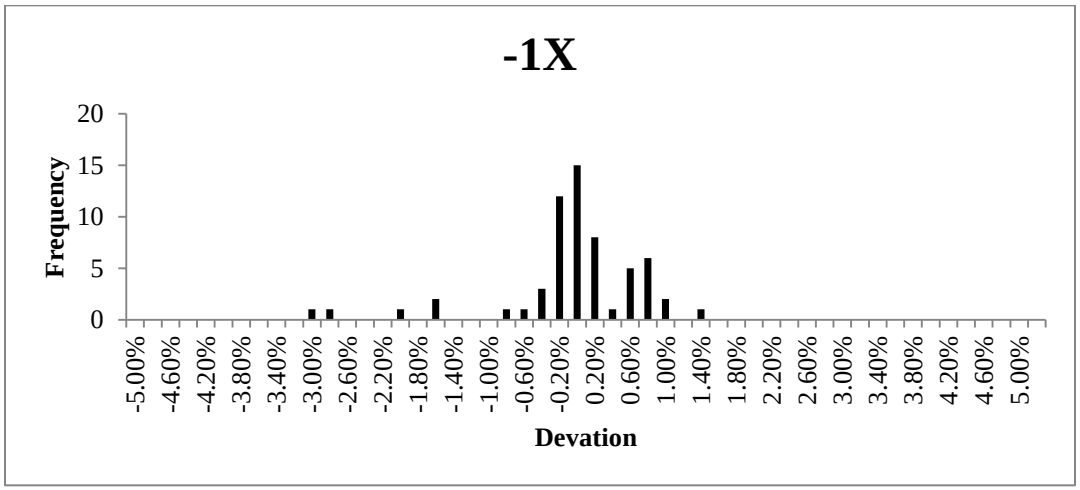


Figure 54: -2x S&P 500 Tracking Fund's Monthly Excess Return Distribution

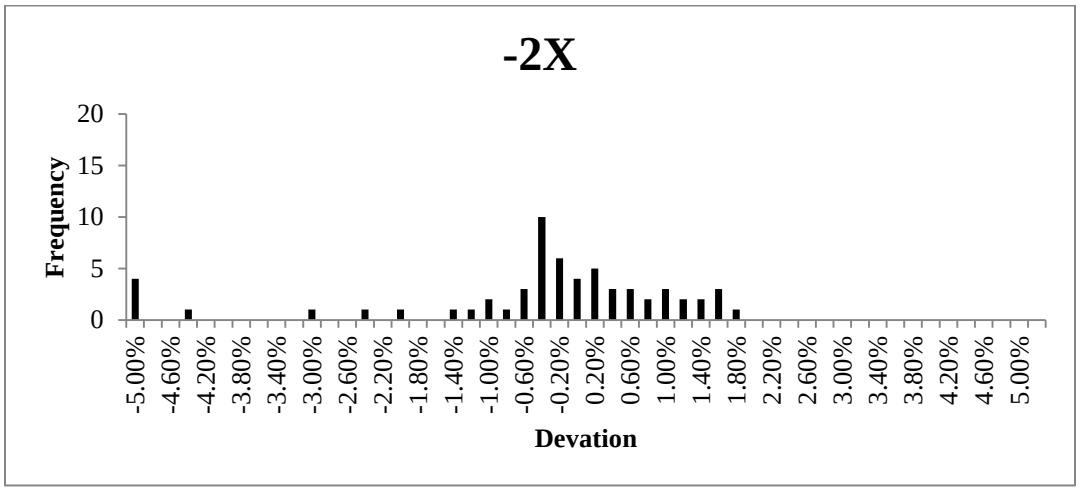


Figure 55: 2x S&P 400 Tracking Fund's Monthly Excess Return Distribution

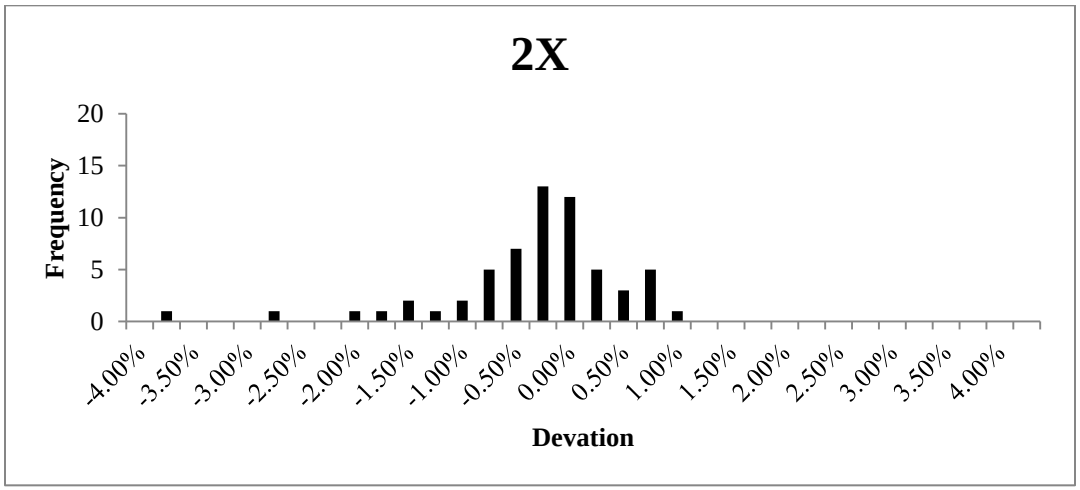


Figure 56: -1x S&P 400 Tracking Fund's Monthly Excess Return Distribution

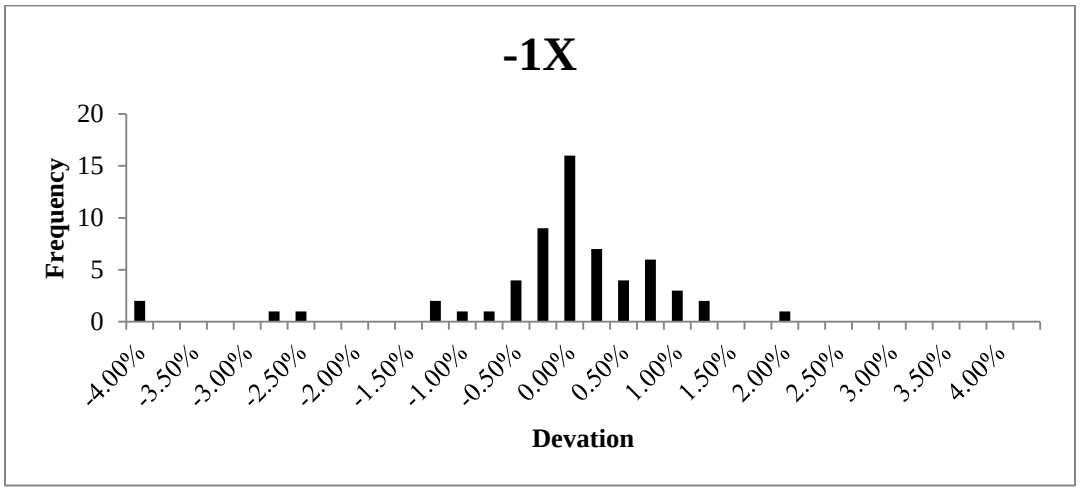


Figure 57: -2x S&P 400 Tracking Fund's Monthly Excess Return Distribution

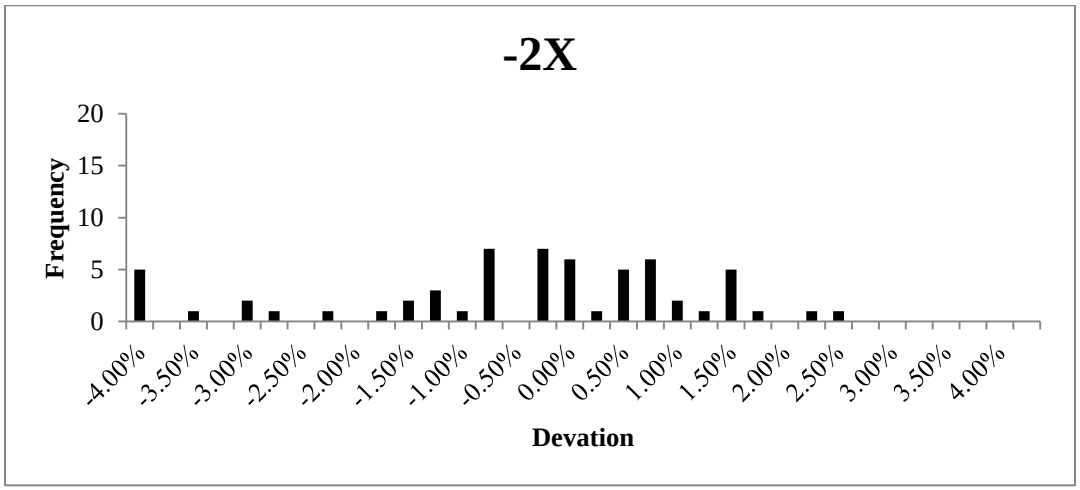


Figure 58: 2x S&P 600 Tracking Fund's Monthly Excess Return Distribution

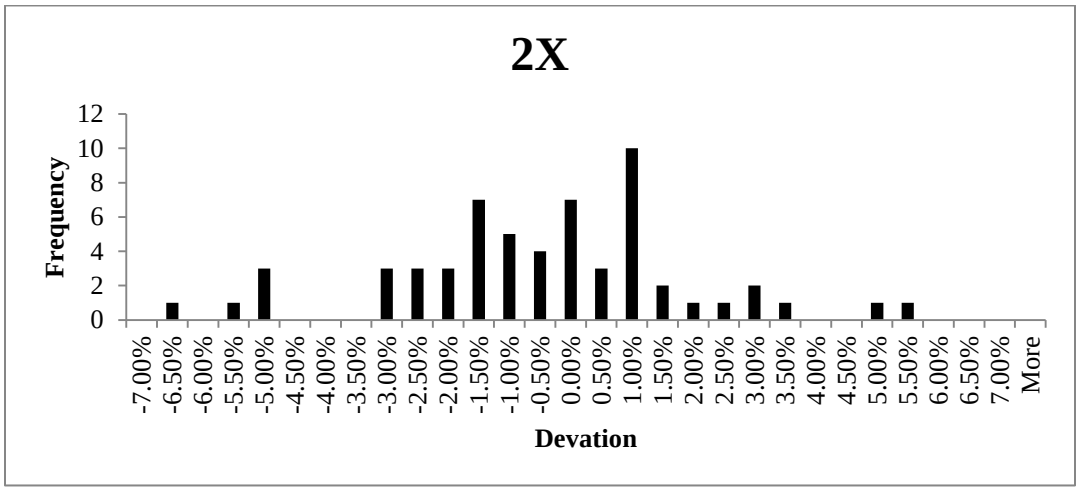


Figure 59: -1x S&P 600 Tracking Fund's Monthly Excess Return Distribution

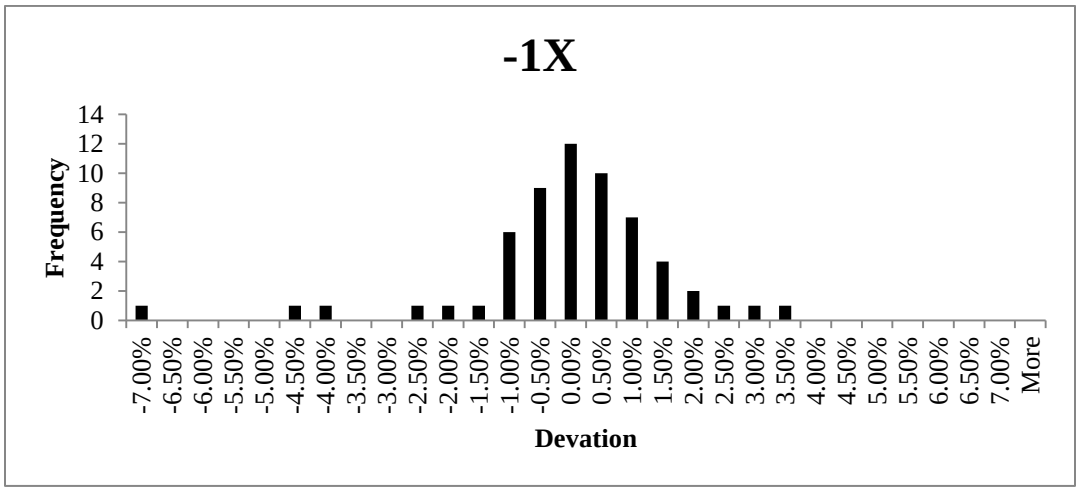
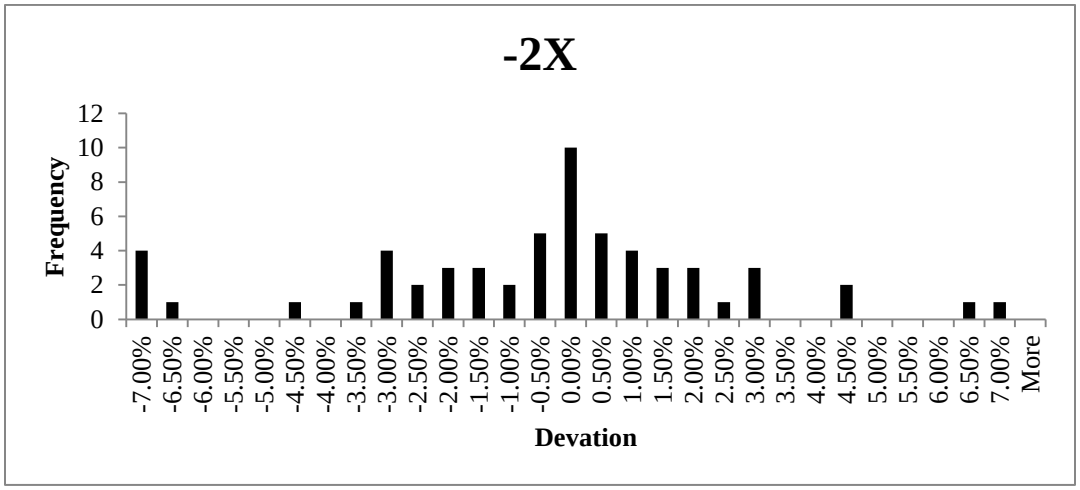


Figure 60: -2x S&P 600 Tracking Fund's Monthly Excess Return Distribution



Looking at the monthly graphs, the distributions seem to have widened a little. The deviations seem to be falling between negative one and a half percent and one and a half percent for the S&P 500 and S&P 400 tracking funds. The S&P 600 tracking funds seem to continue to fall between negative three and three percent, although the distribution is becoming flatter. All of the distributions seem to becoming less normal as holding period increases

Table 18: Monthly Excess Return Skewness

Monthly Excess Return Skewness		
Fund	Skewness	P-Value
S&P 500		
2x	-1.3985	0.0001
3x	-1.5581	0.0009
-1x	-1.6982	0.0000
-2x	-3.1351	0.0000
S&P 400		
2x	-1.6086	0.0000
-1x	-5.4269	0.0000
-2x	-2.1828	0.0000
S&P 600		
2x	-0.1008	0.7285
-1x	-6.5793	0.0000
-2x	-1.0787	0.0013

Looking at the monthly distributions, I find that all of the excess returns except for the 2x S&P 600 tracking fund are skewed. They all have a negative skewness, showing that investors are more likely to see excess returns that are more than the mean.

Chapter Five: Conclusions

The S&P 500 tracking funds had the best performance of the Proshares Leveraged and Inverse S&P tracking ETFs. The daily, weekly, and monthly tracking error all were lower than that of the S&P 400 and S&P 600 tracking funds, as shown in table 16 below.

Table 19: Comparison of Excess Returns

Fund Excess Returns									
Fund	Daily Excess Return	Min	Max	Weekly Excess Return	Min	Max	Monthly Excess Return	Min	Max
S&P 500 Funds									
2x	0.43%	-6.65%	3.12%	0.54%	-4.09%	5.55%	0.74%	-3.06%	1.04%
3x	0.34%	-3.47%	3.16%	0.44%	-3.11%	1.22%	1.11%	-4.60%	1.95%
-1x	0.24%	-2.10%	3.70%	0.33%	-2.26%	2.16%	0.81%	-3.16%	1.33%
-2x	0.39%	-4.06%	6.61%	0.80%	-5.85%	4.70%	2.47%	-13.28%	1.64%
S&P 400 Funds									
2x	0.60%	-5.92%	5.12%	0.68%	-5.08%	4.68%	0.81%	-3.80%	0.88%
-1x	0.58%	-14.49%	3.93%	1.06%	-13.95%	4.40%	2.23%	-15.47%	1.97%
-2x	0.56%	-5.03%	5.20%	1.30%	-8.34%	9.80%	2.58%	-12.01%	2.39%
S&P 600 Funds									
2x	3.15%	-75.72%	46.57%	2.29%	-12.09%	10.95%	2.35%	-6.56%	5.32%
-1x	1.85%	-33.49%	37.47%	2.47%	-35.77%	6.08%	4.87%	-36.00%	3.19%
-2x	3.11%	-46.36%	75.19%	2.19%	-12.13%	10.30%	3.58%	-13.66%	6.84%

All of the S&P 500 (large-cap stock) funds had daily excess returns that are not statistically different from zero at the 95% confidence level. This tells us that they did not produce any higher or lower returns than expected given a one day holding period. However, they all had betas that are statistically different than one at the 95% confidence level. With their betas all being smaller than one, investors are actually taking on less systematic (benchmark) risk than the fund states.

The S&P 400 (mid-cap stock) tracking funds also track well on a daily basis, producing relatively small tracking error (although larger than the S&P 500 tracking funds), and no statistically significant excess return. The betas are also significantly less than one. Again, the investor takes on less risk than what the fund states for a daily holding period.

The S&P 600 (small-cap stock) tracking funds do not track as well, producing larger tracking errors. Although their alphas are zero at the 95% confidence level, meaning they do not produce any more excess return than they should, their betas are not one. Again, this results in the investors taking on less risk than the fund states with betas far less than one. The r-squareds of the regressions of the S&P 600 tracking funds (on index multiples) are far less than one, suggesting the funds do a relatively poor job of following the appropriate multiple of the index.

A comparison of the tracking error of all leverage and inverse ETFs studied is in table 17 below. This measure is the standard deviation of the fund excess return (fund return minus the expected return (index return times the appropriate multiple)) over the full sample period.

Table 20: Funds' Tracking Error Over Time

Tracking Error			
Fund	**Daily**	**Weekly**	**Monthly**
S&P 500 Funds			
2x	0.43%	0.54%	0.74%
3x	0.34%	0.44%	1.11%
-1x	0.24%	0.33%	0.81%
-2	0.39%	0.80%	2.47%
S&P 400 Funds			
2x	0.60%	0.68%	0.81%
-1x	0.58%	1.06%	2.23%
-2x	0.56%	1.30%	2.58%
S&P 600 Funds			
2x	3.15%	2.29%	2.35%
-1x	1.85%	2.47%	4.87%
-2x	3.11%	2.19%	3.58%

Looking at the weekly and monthly data, I see that, on average, tracking error increases with return horizon. That is, the longer the holding period of the leveraged and inverse funds, the larger the tracking error, on average. There is an explanation for why these funds track weaker in longer holding periods. Consider the example in table 21 below.

Table 21: Hypothetical Two Day Holding Period

2 Day Holding Period			
	Day 1	Day 2	Overall
Underlying Index	10.00%	10.00%	21.00%
Leveraged ETF (With Compounding)	20.00%	20.00%	44.00%
2x Underlying Index Return	20.00%	20.00%	42.00%

The returns of the underlying index are multiplied by two-the equivalent to what a 2x leveraged fund should provide. If the underlying index returns 10% a day, the leveraged ETF

should return twice that, or 20%. Compounding the returns of the underlying index gives 21% after two days. Compounding the returns from the leveraged ETF gives 44%. This is different (by 2%) from multiplying the two day holding period return of the underlying index (21%) by two, giving 42%. From this, I see that, over a longer holding period, the leveraged ETF will have lower returns than it promises, even if it does what it promises on a daily basis. The effects get worse with longer holding periods, as shown in table 22 below.

Table 22: Hypothetical 5-Day Holding Period (High Volatility)

5-Day Holding Period						
	Day 1	Day 2	Day 3	Day 4	Day 5	Overall
Index Return	10.00%	10.00%	10.00%	10.00%	10.00%	61.05%
2x Leveraged ETF (With Compounding)	20.00%	20.00%	20.00%	20.00%	20.00%	148.83%
2x Underlying Index Return	20.00%	20.00%	20.00%	20.00%	20.00%	122.10%

If the holding period is five days, the deviation grows larger, to 26.73%. This demonstrates that leveraged ETFs will over perform over longer holding periods, even if the fund does exactly what is promised on a daily basis. The longer the holding period, the bigger the effect is.

The effects of volatility can be displayed here as well. See table 23 below.

Table 23: Hypothetical Five Day Holding Period (Low Volatility)

5-Day Holding Period						
	Day 1	Day 2	Day 3	Day 4	Day 5	Overall
Index Return	1.00%	1.00%	1.00%	1.00%	1.00%	5.10%
2x Leveraged ETF (With Compounding)	2.00%	2.00%	2.00%	2.00%	2.00%	10.41%
2x Underlying Index Return	2.00%	2.00%	2.00%	2.00%	2.00%	10.20%

If the daily returns are reduced to one percent, instead of our previous example of ten percent, the effects of the compounding are far less. Given the same five day holding period, the deviation is only 0.21% in our low market volatility example. This shows that volatility can have a significant effect on the deviations of the returns over longer holding periods. This is a possible explanation for the tracking ability of the S&P 600 tracking funds. Small-cap funds tend to have higher levels of volatility compared to large-cap and mid-cap funds.

Comparing the tracking ability of the funds by their amount of leverage, the single inverse (-1x) funds and the double (2x) funds seem to track better than the double inverse (-2x) funds. Having only one triple (3x) fund, it is not included in this analysis as there is nothing to compare it to.

If the tracking ability of funds is compared by regression model fit, there are different results. The funds with the best overall model fit are the -2x funds, followed by the 2x funds, and finally the -1x funds. Here, the funds are not affected by bear or bull position. They are, however, affected by leverage. The 2x funds (both bull and bear) tracked better than the -1x fund. Looking only at the S&P 500 funds to make a comparison with a 3x fund, reinforces the previous statement that leverage affects the model fit. The 3x fund had the best model fit of any fund analyzed. From this perspective, leverage seems to enhance model fit for the funds returns.

Comparing the skewness of returns as shown graphically in this paper, it seems the holding period has an effect. The longer the holding period, the more likely it is to see a negative skewness in the excess return distribution. That tells us the funds are more likely to have negative excess returns that are larger than the mean.

Finally, examining Figures 1-30, I see that there are greater excess return levels in times of very high market volatility. During the time of crisis, excess return levels become more extreme relative to the rest of the period studied. The largest excess return values are positioned around the area of autumn of 2008, when the housing crisis was taking its toll on the markets. This shows that extreme volatility in the market disrupts the tracking ability of all leveraged funds, possibly because of market failure due to lack of liquidity.

Overall, looking at the funds from the perspective of market capitalization of the underlying assets, it seems the large cap funds track the best overall, with the smallest range of tracking errors, and best fitting models in every category. The small cap funds seem to track the worst with the largest amounts of tracking error and models that don't seem to fit the data well. The mid cap funds come a lot closer to the large cap funds in ability to track, with a relatively close range of tracking error and fit of models.

To conclude, market capitalization seems to play a large role in the tracking ability of these funds. The larger the market capitalization of the underlying stocks, the better they track. All of the funds perform better in periods of low market volatility and have larger excess returns on days of large market swings. The amount of leverage does not seem to have a concrete effect on the tracking ability of the funds, producing different results from different perspectives. Holding period has a definite effect on the tracking ability of the funds, although this can be explained. Longer holding periods are correlated with higher levels of tracking error in the funds.

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